Consumer Preferences and Choice

Lecture 1

Div Bhagia

Introduction

Why Study Microeconomics?

- **Non-economists typically think of economics when it concerns the *macro* stuff** (unemployment, inflation, growth, recessions etc.)

- **However, the macro economy is made up of millions of *micro* decisions.**
  - **People:** what to buy, how much to work, how much to save
  - **Firms:** pricing, hiring, location, investment
  - **Government:** infrastructure, regulation, tax policy

- **Markets coordinate these decisions invisibly and *often* effectively**.
  - Gas is at the pump when you need it
  - Jobs exist for qualified workers
  - Products get delivered

Why Study Microeconomics?

- **However, sometimes markets produce undesirable outcomes or fail.**
  - Each micro failure may seem small (one shortage, one person unemployed, one overpriced good)
  - But small failures add up to large macro consequences

- **In this course, we will study:**
  - How individuals and firms make decisions?
  - How markets coordinate these decisions?
  - When markets work efficiently and when they fail?
  - What interventions can help?

Course Roadmap

Competitive Markets

- Consumer choice (1-2)
- Firm production (3)
- Market equilibrium (4)
- Efficiency and welfare (5)

Market Power

- Monopoly and oligopoly (6-7)
- Labor markets and monopsony (8)

Market Failures

- Information asymmetries (9)
- Externalities and public goods (10)

Uncertainty & Strategy

- Decisions under uncertainty (11)
- Game theory (12)

Today's Lecture

Central question: How do individuals make choices?

Economic Approach to Choice

- Individuals have **preferences** over outcomes
- These preferences are **rational** (satisfy certain axioms)
- We can represent preferences with a **utility function**
- Given constraints, individuals **maximize utility**

Does everyone really "maximize utility"?

- Maybe not consciously—but people behave "as if" they do
- The model predicts behavior relatively well
- Alternative: behavioral economics

Preferences and Utility

Preferences: Basic Setup

We consider a consumer choosing between bundles of goods.

- **Consumption bundle**: (x, y) where $x, y \geq 0$
- **Preference relation**: $\succeq$ (weakly preferred to)
  - $A \succeq B$: Bundle A is at least as good as bundle B
  - $A \succ B$: Bundle A is strictly preferred to B
  - $A \sim B$: Consumer is indifferent between A and B

Key question: What properties should preferences satisfy for them to be "rational"?

Axioms of Rational Choice

1. Completeness

For any two bundles A and B, the consumer can state which is preferred or that they are indifferent:

$$A \succeq B, \quad B \succeq A, \quad \text{or both (indifference)}$$

Interpretation: Consumers can always make comparisons. Rules out indecision.

2. Transitivity

If $A \succeq B$ and $B \succeq C$, then $A \succeq C$

Interpretation: Preferences are internally consistent. No cycles.

Axioms of Rational Choice (cont.)

3. Continuity

Small changes in consumption bundles lead to small changes in preferences.

Technical: For any bundle A, the sets $\{B : B \geq A\}$ and $\{B : A \geq B\}$ are closed.

Interpretation: No sudden jumps. Preferences are "smooth."

4. Non-satiation (Monotonicity)

More is better: If A has at least as much of everything as B, and strictly more of at least one good, then $A > B$.

Interpretation: Consumers always prefer more to less (at least weakly).

Axioms of Rational Choice (cont.)

5. Convexity

Averages are preferred to extremes. If $A \sim B$, then:

$$\lambda A + (1 - \lambda)B \succeq A \text{ for } \lambda \in [0, 1]$$

Interpretation: Consumers prefer balanced consumption bundles. Diminishing marginal rate of substitution.

Example: If you're indifferent between (6 apples, 0 oranges) and (0 apples, 6 oranges), you prefer (3 apples, 3 oranges) to either extreme.

When Do Axioms Fail?

Behavioral Economics Violations

- **Framing effects**: Preferences change based on how options are presented
- **Intransitivity**: Preference reversals in complex choices
- **Present bias**: Time-inconsistent preferences
- **Reference dependence**: Preferences depend on current endowment (loss aversion)
- **Bounded rationality**
  - Too many options → choice paralysis
  - Computational constraints
  - Limited attention

From Preferences to Utility

Key Theorem: If preferences satisfy completeness, transitivity, continuity, and monotonicity, then there exists a continuous **utility function** $U(x, y)$ that represents them:

$$A \succeq B \iff U(A) \geq U(B)$$

Interpretation: We can assign numbers to bundles such that higher numbers = more preferred.

Important: Utility is **ordinal**, not **cardinal**

- Only the **ranking** matters, the magnitude of utility has no meaning.
- $U(A) = 10, U(B) = 5$ tells us $A \succ B$. It does NOT mean "A is twice as good as B"

Monotonic Transformations

- Since utility is ordinal, we can apply any **strictly increasing transformation** without changing preferences:
- If $U(x, y)$ represents preferences, so does $V(x, y) = f(U(x, y))$ for any strictly increasing f.
- **Examples**:
  - $U(x, y) = xy$ and $V(x, y) = \ln(xy)$ represent the same preferences
  - $U(x, y) = x^{0.5}y^{0.5}$ and $V(x, y) = xy$ represent the same preferences

Why this matters: We can transform utility functions to make calculations easier.

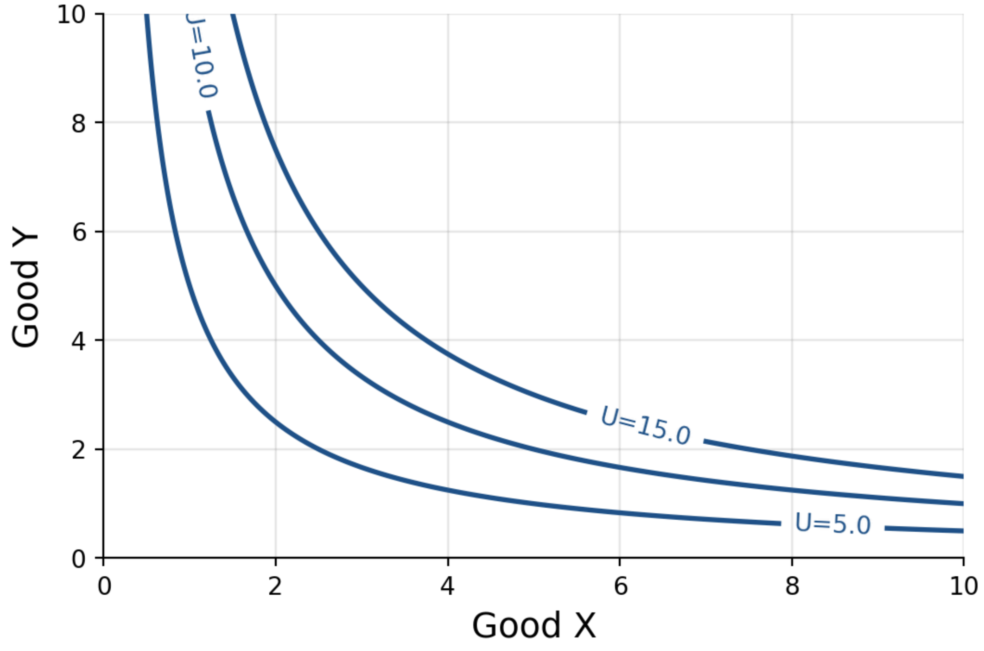
Indifference Curves & MRS

Indifference Curves

An **indifference curve** is the set of all bundles that give the same utility level:

$$IC(U_0) = \{(x, y) : U(x, y) = U_0\}$$

Interpretation: The consumer is indifferent between any two points on the same curve.



Indifference Curves for U(x,y) = xy

Properties of Indifference Curves

Under our axioms, indifference curves must be:

1. **Downward sloping** (from non-satiation)
   - To keep utility constant, if x increases, y must decrease
2. **Do not cross** (from transitivity)
   - If they crossed, we'd have $A \sim B$ and $A \sim C$ but $B \nsim C$
3. **Convex to the origin** (from convexity of preferences)
   - Averages preferred to extremes
   - Equivalently: diminishing marginal rate of substitution
4. **Higher curves represent higher utility** (from monotonicity)

Marginal Rate of Substitution (MRS)

- The **marginal rate of substitution** is the rate at which the consumer is willing to trade good Y for good X while maintaining constant utility.
- **Geometrically**: MRS = -(slope of indifference curve)

$$\mathrm{MRS} = -\frac{dy}{dx}\bigg|_{U=\mathrm{const}}$$

- **Interpretation**: How many units of Y are you willing to give up to get one more unit of X?
- **Example**: If MRS = 2, you're willing to give up 2 units of Y to get 1 more unit of X (and remain indifferent).

Deriving the MRS Formula

Along an indifference curve, utility is constant: $U(x, y) = \bar{U}$. Taking the total differential:

$$dU = \frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy = 0$$

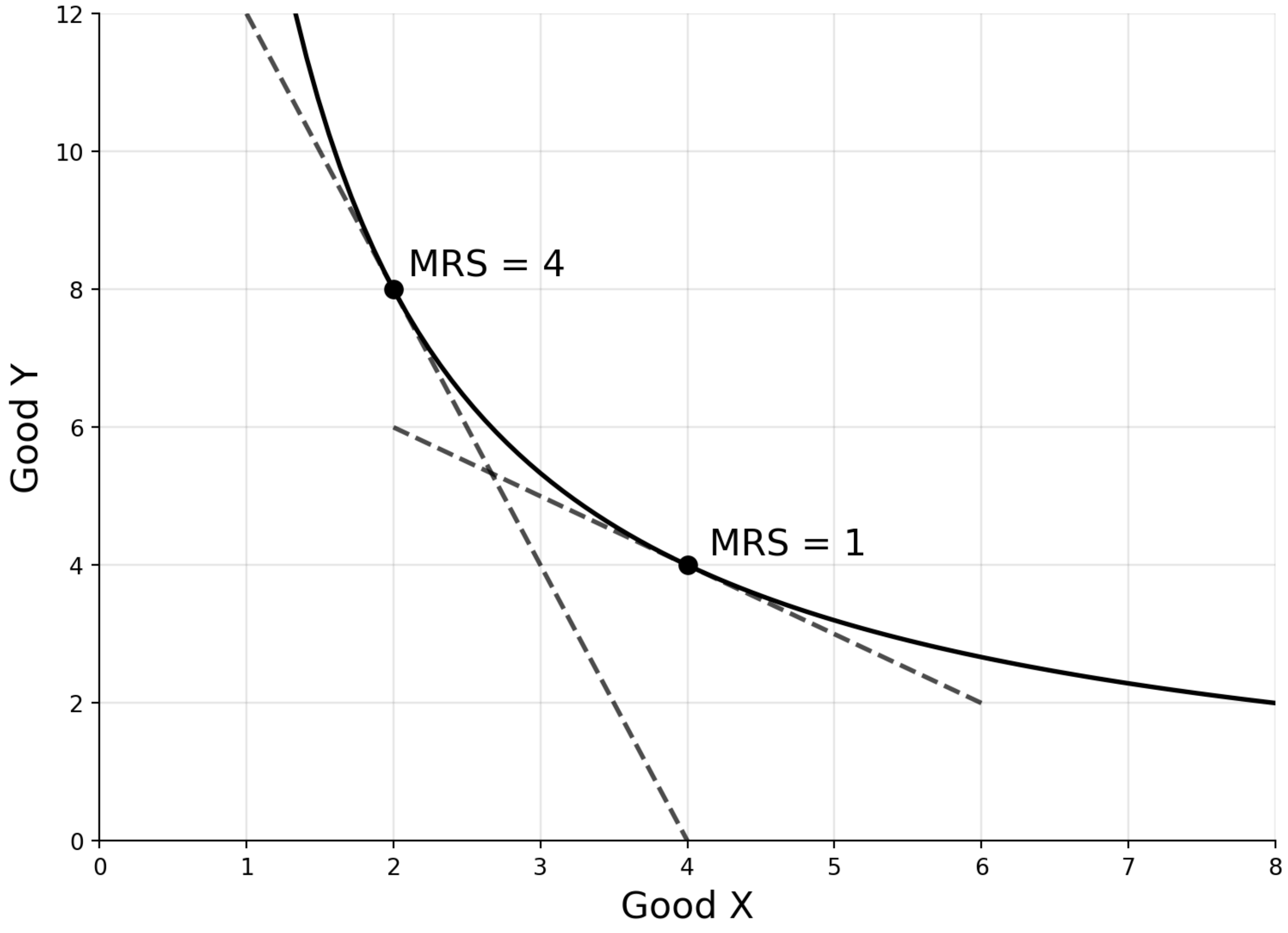
Rearranging:

$$\frac{\partial U}{\partial y}dy = -\frac{\partial U}{\partial x}dx \quad \rightarrow \quad \frac{dy}{dx} = -\frac{\partial U/\partial x}{\partial U/\partial y} = -\frac{MU_x}{MU_y}$$

Therefore:

$$\boxed{MRS = -\frac{dy}{dx} = \frac{MU_x}{MU_y}}$$

Visualizing MRS



Diminishing MRS

Convexity assumption → Diminishing MRS

As you consume more of good X (moving right along an IC), the MRS decreases:

- When you have a lot of Y and little X: High MRS (willing to give up a lot of Y for more X)
- When you have a lot of X and little Y: Low MRS (not willing to give up much Y for more X)

Economic intuition: Scarcity increases value. The less you have of something, the more you value additional units.

Common Utility Functions

Perfect Substitutes

Goods that can be substituted at a constant rate.

$$U(x, y) = ax + by$$

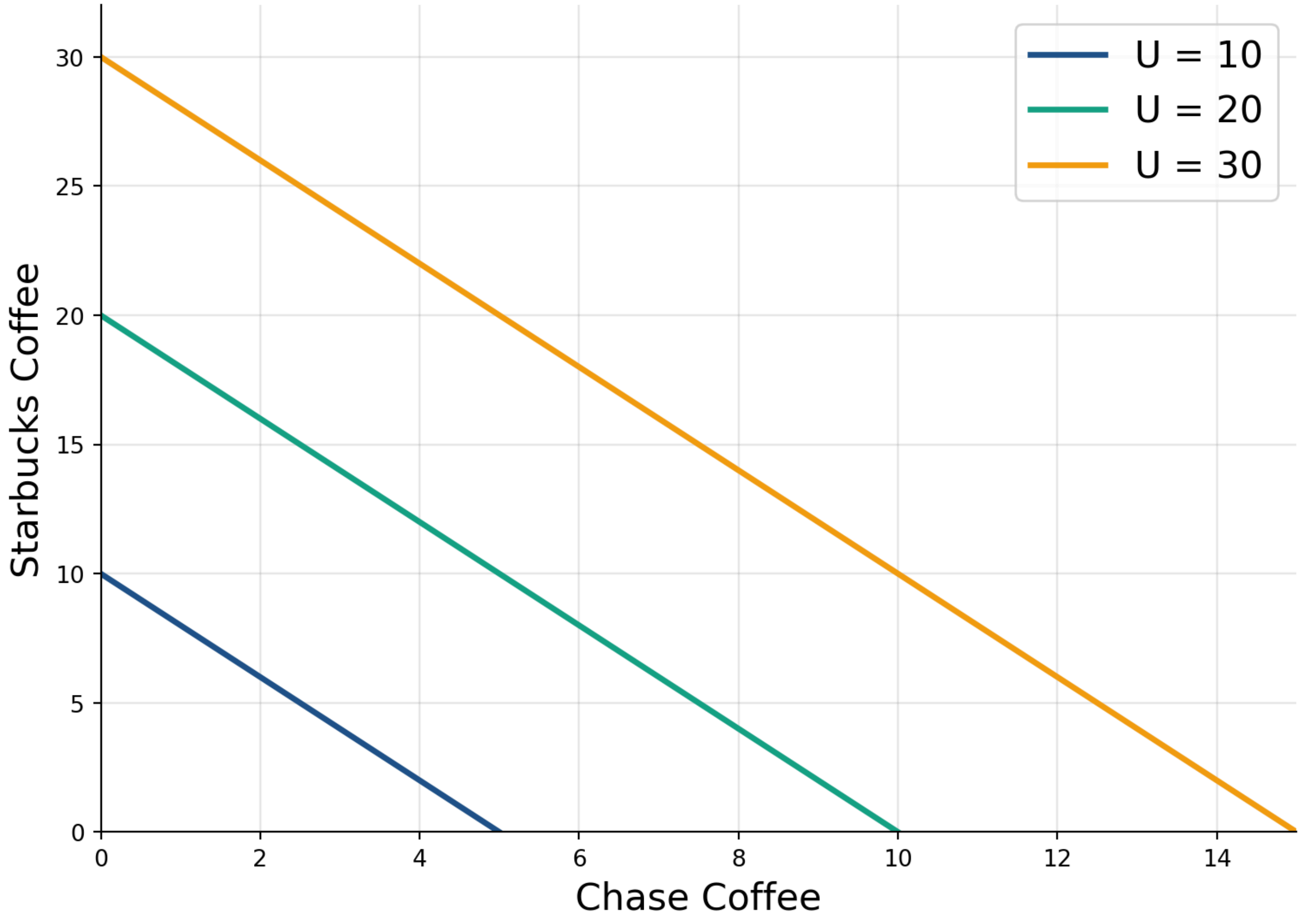
Examples:

- Different brands of the same product (e.g., Coke vs Pepsi for some)
- Coffee from different cafes
- Generic vs brand-name drugs (if truly equivalent)

Key features:

- Indifference curves are **straight lines**
- MRS is **constant**: $MRS = a/b$
- Consumer willing to trade at fixed rate regardless of bundle

Perfect Substitutes Graph



Perfect Substitutes: U = 2x + y

Perfect Complements

Goods that must be consumed in fixed proportions.

$$U(x, y) = \min\{ax, by\}$$

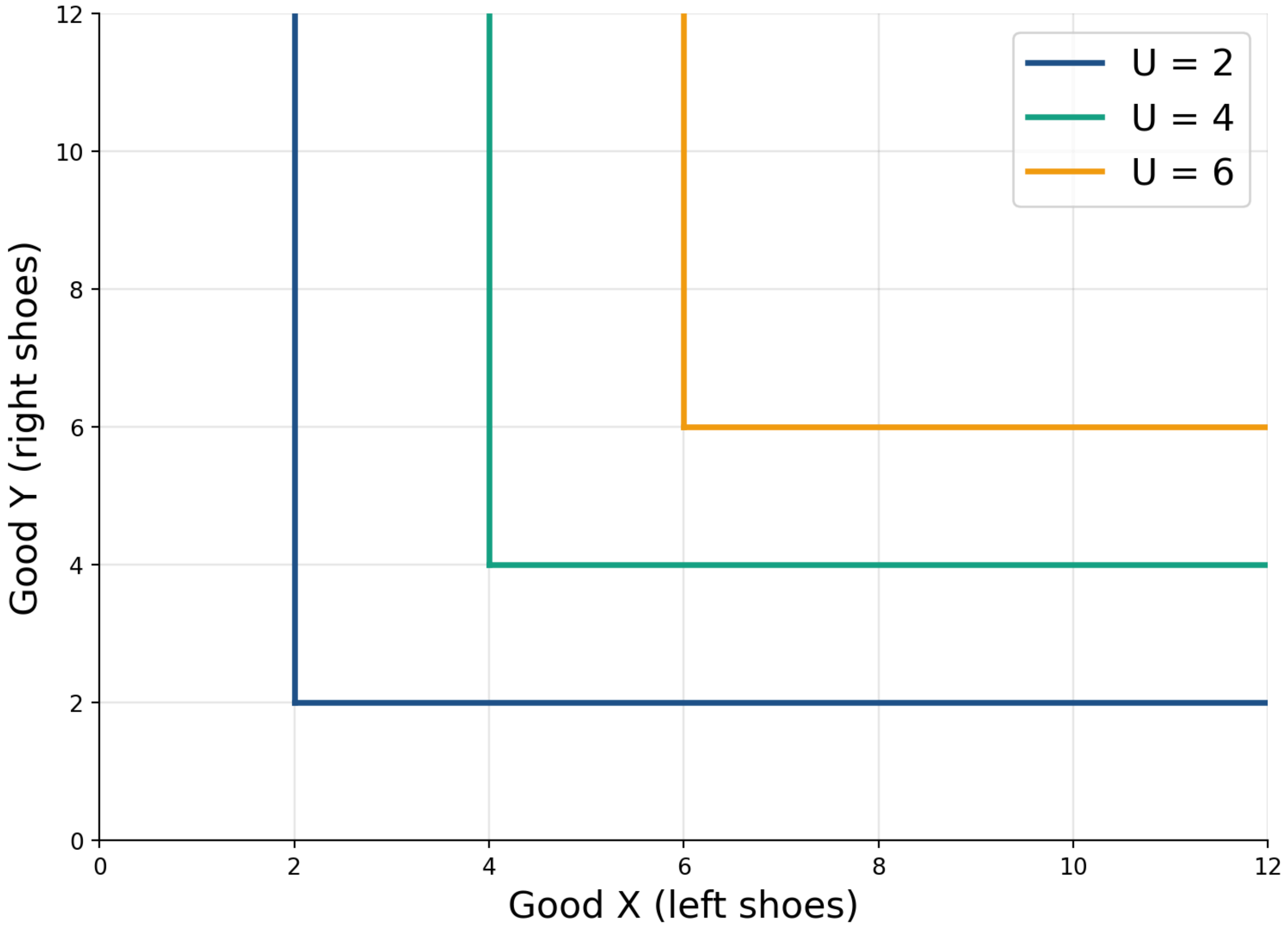
Examples:

- Left and right shoes
- Cars and tires (need 4 tires per car)
- Computers and monitors

Key features:

- Indifference curves are **L-shaped**
- Consumed in fixed ratio: $x/y = b/a$
- MRS is undefined (technically infinite or zero, depending on side)

Perfect Complements Graph



Perfect Complements: U = min{x, y}

Cobb-Douglas Utility

The most widely used functional form in economics:

$$U(x, y) = x^{\alpha} y^{\beta}$$

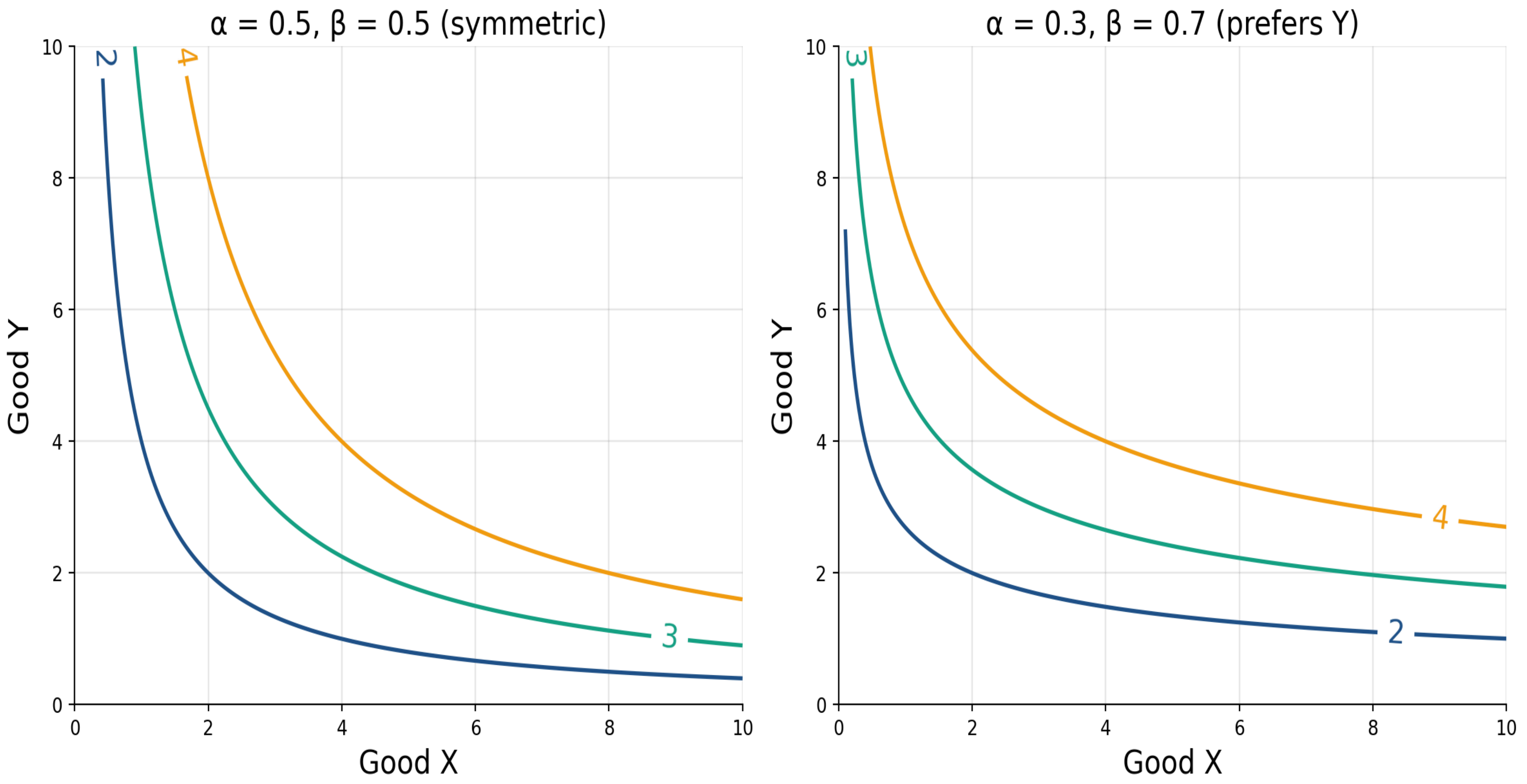
Or equivalently (applying monotonic transformation):

$$U(x, y) = \alpha \ln x + \beta \ln y$$

Key features:

- Smooth, convex indifference curves
- Interior solutions (typically)
- Constant expenditure shares
- $\text{MRS} = \frac{\alpha}{\beta} \cdot \frac{y}{x}$

Cobb-Douglas Graph



Cobb-Douglas Utility: $U = x^{\alpha}y^{\beta}$

Cobb-Douglas: MRS Calculation

For $U(x, y) = x^{\alpha} y^{\beta}$:

Step 1: Find marginal utilities

$$MU_x = \frac{\partial U}{\partial x} = \alpha x^{\alpha - 1} y^{\beta}, \qquad MU_y = \frac{\partial U}{\partial y} = \beta x^{\alpha} y^{\beta - 1}$$

Step 2: Calculate MRS

$$MRS = \frac{MU_x}{MU_y} = \frac{\alpha x^{\alpha - 1} y^{\beta}}{\beta x^{\alpha} y^{\beta - 1}} = \frac{\alpha}{\beta} \cdot \frac{y}{x}$$

MRS depends on the ratio y/x and the preference parameters α/β.

CES Utility

Constant Elasticity of Substitution (CES) utility function:

$$U(x,y) = (ax^{\varrho} + by^{\varrho})^{1/\varrho}, \qquad \varrho \leq 1, \varrho \neq 0$$

Elasticity of substitution: $\sigma = \frac{1}{1-\varrho}$

Special cases:

- $\varrho \to -\infty$: Perfect complements ($\sigma = 0$)
- $\varrho = 0$: Cobb-Douglas ($\sigma = 1$)
- $\varrho = 1$: Perfect substitutes ($\sigma = \infty$)

Flexibility: CES nests all three cases mentioned.

Utility Maximization

The Budget Constraint

Consumers have limited income I and face prices p_x, p_y for goods: $p_x \cdot x + p_y \cdot y \leq I$

Budget line: Set of bundles that cost exactly I:

$$p_x \cdot x + p_y \cdot y = I$$

Rearranging for y:

$$y = \frac{I}{p_y} - \frac{p_x}{p_y} x$$

- **Intercept**: I/p_y (max amount of y if $x = 0$)
- **Slope**: $-p_x/p_y$ (opportunity cost of x in terms of y)

Budget Constraint Graph

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/var/folders/m0/81ww_p5n651_dc02g0cwpk2w0000gq/T/ipykernel_41558/2446795308.py:15: UserWarning:

color is redundantly defined by the 'color' keyword argument and the fmt string "b-" (-> color='b'). The keyword
argument will take precedence.
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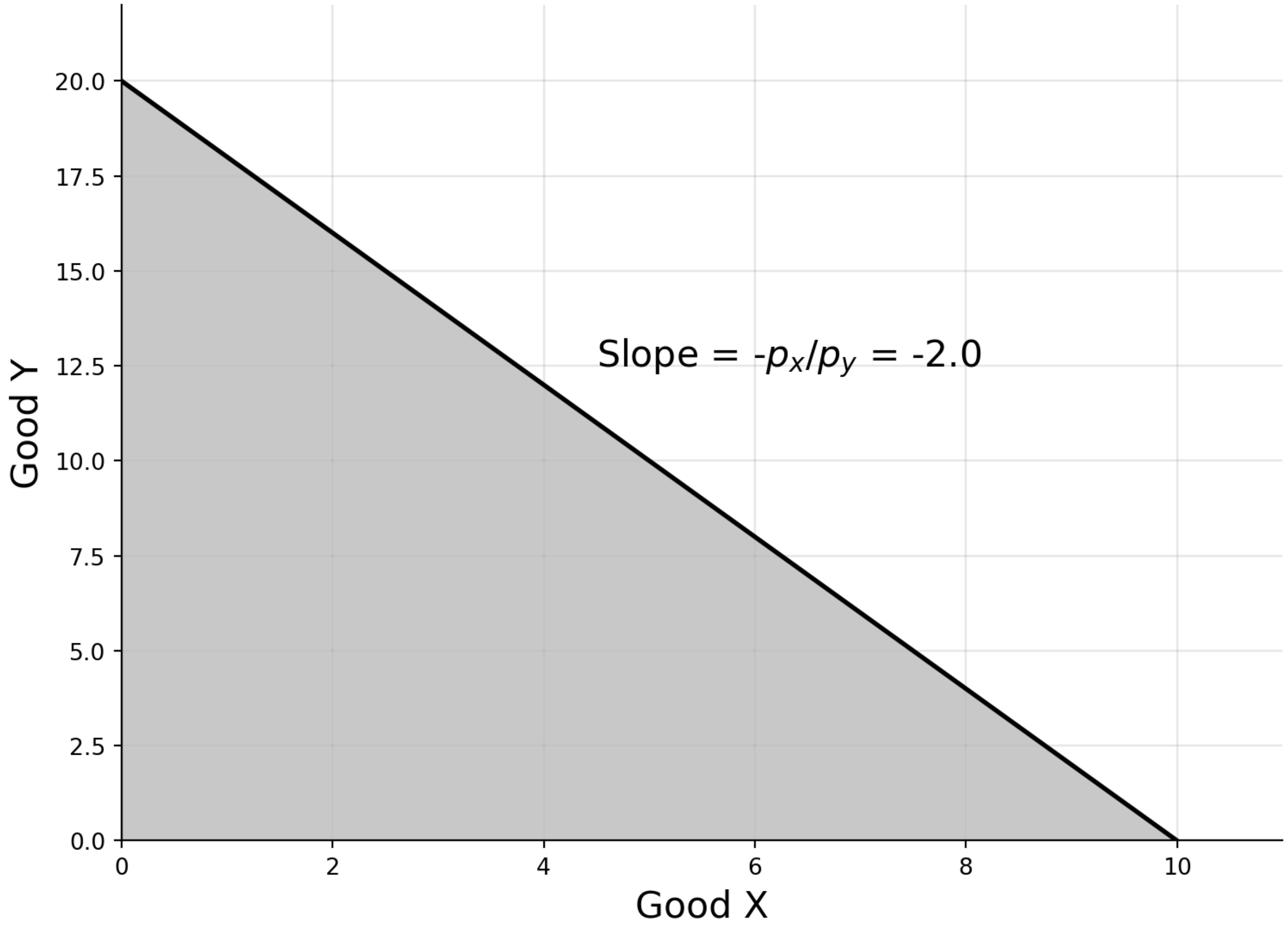


The Consumer's Problem

The consumer chooses (x, y) to:

$$\max_{x,y} \quad U(x, y)$$

subject to:

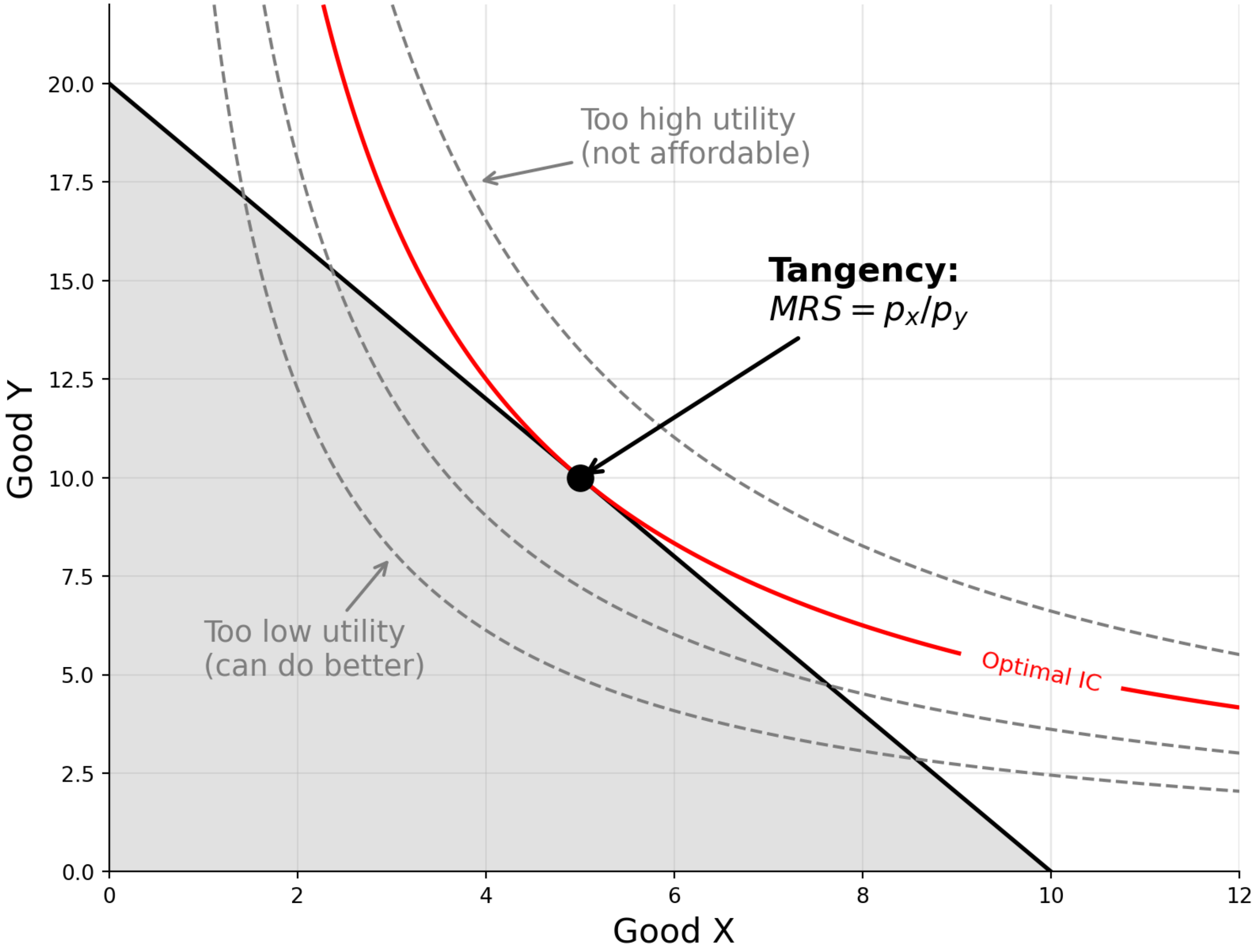
$$p_x x + p_y y = I$$

$$x \geq 0, \quad y \geq 0$$

Goal: Find the highest indifference curve that touches the budget line.

Intuition: Get as much utility as possible given your budget.

Graphical Solution



Why Tangency is Optimal

At the tangency point: MRS = p_x/p_y

Intuition: Consumer's subjective tradeoff (MRS) equals market tradeoff

- **MRS**: How much Y you're willing to give up for 1 unit of X
- p_x/p_y: How much Y you must give up (in market) for 1 unit of X

If MRS > p_x/p_y:

- You value X more than market does
- Should buy more X, less Y

If MRS < p_x/p_y:

- You value X less than market does
- Should buy less X, more Y

The Lagrangian Method

The consumer's problem:

$$\max_{x,y} \quad U(x,y) \quad \text{subject to} \quad p_x x + p_y y = I$$

Denote λ as the **Lagrange multiplier** and setup the **Lagrangian**:

$$L(x,y,\lambda) = U(x,y) + \lambda(I - p_x x - p_y y)$$

First-order conditions (FOCs):

1. $\frac{\partial L}{\partial x} = \frac{\partial U}{\partial x} - \lambda p_x = 0$
2. $\frac{\partial L}{\partial y} = \frac{\partial U}{\partial y} - \lambda p_y = 0$
3. $\frac{\partial L}{\partial \lambda} = I - p_x x - p_y y = 0$

First-Order Conditions

From the first two FOCs:

$$MU_x = \lambda p_x \quad \text{and} \quad MU_y = \lambda p_y$$

Dividing these:

$$\frac{MU_x}{MU_y} = \frac{p_x}{p_y}$$

This is exactly the tangency condition: **MRS = price ratio**!

Solving the system of equations given by the FOCs yields the optimal consumption bundle (x^*, y^*) and the multiplier λ^*.

Interpretation of λ

From FOCs: $\lambda = \frac{MU_x}{p_x} = \frac{MU_y}{p_y}$

λ = marginal utility of income

- How much utility increases if income increases by $1
- Measures the "value" of relaxing the budget constraint
- Important for welfare analysis

Example: If $\lambda = 0.5$:

- $1 more of income increases utility by 0.5 utils
- Equivalently: consumer willing to pay $2 for 1 more util

Note: λ decreases as income increases (diminishing marginal utility of income)

Example: Cobb-Douglas Utility

Setup: $U(x, y) = x^{\alpha}y^{\beta}$, budget: $p_x x + p_y y = I$

Step 1: Form the Lagrangian

$$L = x^{\alpha}y^{\beta} + \lambda(I - p_x x - p_y y)$$

Step 2: Take FOCs

$$\frac{\partial L}{\partial x} = \alpha x^{\alpha-1} y^{\beta} - \lambda p_x = 0$$

$$\frac{\partial L}{\partial y} = \beta x^{\alpha} y^{\beta-1} - \lambda p_y = 0$$

$$\frac{\partial L}{\partial \lambda} = I - p_x x - p_y y = 0$$

Example: Cobb-Douglas Utility (cont.)

Step 3: Solve the system of equations to find x^*, y^*, and λ^*.

$$x^* = \frac{\alpha I}{(\alpha + \beta)p_x}, \quad y^* = \frac{\beta I}{(\alpha + \beta)p_y}$$

See handout for full derivation.

Key results:

- **Expenditure on X**: $p_x x^* = \frac{\alpha I}{\alpha+\beta}$
- **Expenditure on Y**: $p_y y^* = \frac{\beta I}{\alpha+\beta}$
- **Expenditure shares are constant:** $\frac{\alpha}{\alpha+\beta}$ and $\frac{\beta}{\alpha+\beta}$

Constant Expenditure Shares

Cobb-Douglas utility implies constant expenditure shares regardless of income or prices.

Which of the following goods do you think have roughly constant expenditure shares in real life?

1. Food
2. Housing
3. Travel
4. Charitable giving

Engel's Law: As income rises, the proportion spent on food decreases.

How to model this?

Application: Cash vs In-Kind Transfers

Food Stamps

Policy question: Give \$100 cash or \$100 food stamps?

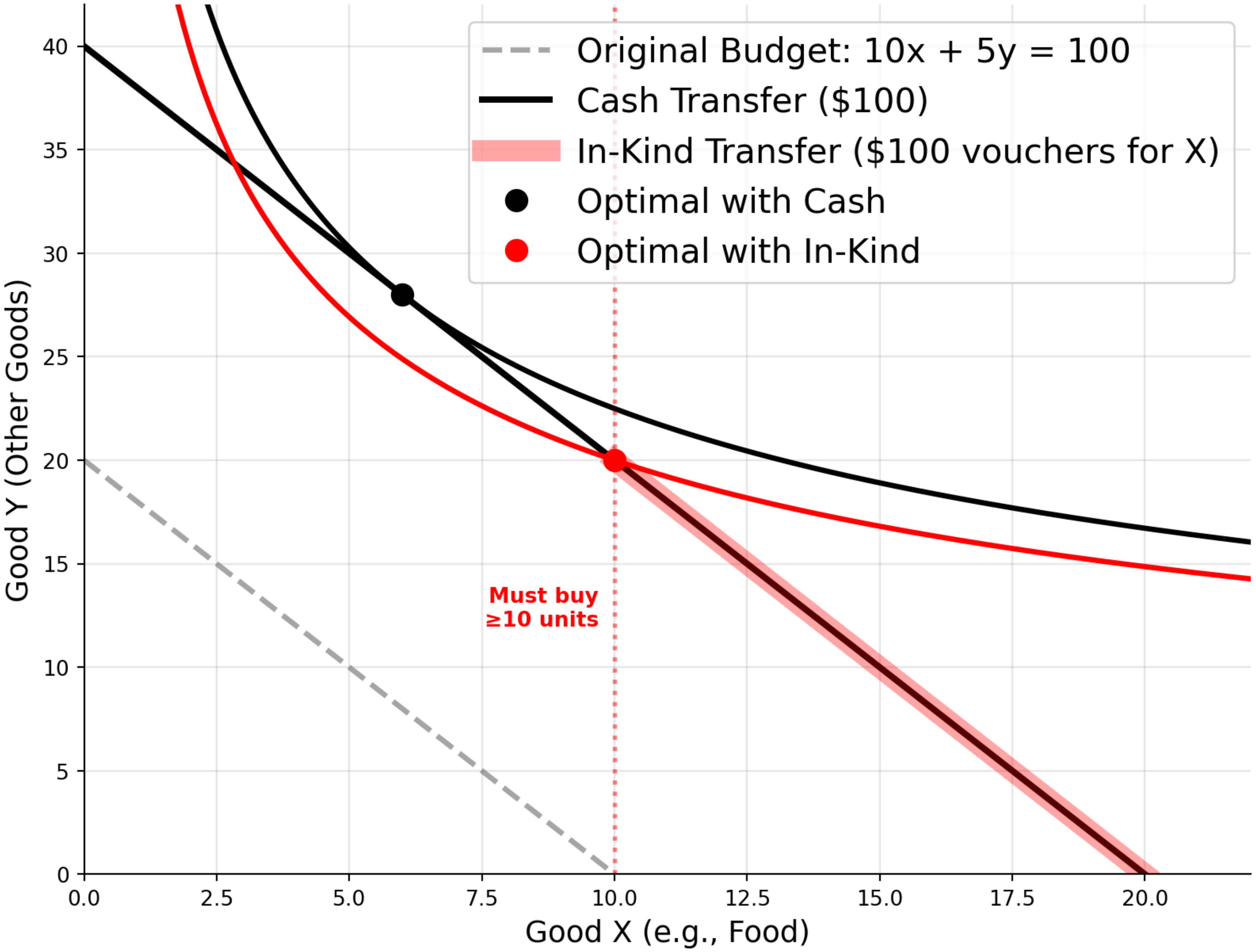
Setup:

- Two goods: food (x) and other goods (y)
- Original income: I

With cash: Budget is $(p_x, p_y, I + 100)$

With food stamps: Can buy up to $100/p_x$ extra food, but must spend at least that on food

Cash vs Food Stamps on a Graph



Cash vs In-Kind Transfers

Key insight: Cash transfers are at least as good as in-kind transfers (eg. SNAP, housing vouchers, energy assistance etc.)

- More flexibility → reach higher indifference curve (or same)
- If in-kind doesn't bind, cash and in-kind are equivalent
- If in-kind binds, cash is strictly better

Then why use in-kind transfers?

- Paternalism (society values food/housing more than recipients)
- Political economy (easier to justify to voters)
- Externalities (e.g., nutrition benefits for children)

Summary

What we covered:

1. **Preferences**: Axioms of rational choice, utility representation
2. **Indifference curves**: Properties, MRS
3. **Common utility functions**: Perfect substitutes, perfect complements, Cobb-Douglas, CES
4. **Utility maximization**: Tangency condition (MRS = p_x/p_y), Lagrange method, Indirect utility
5. **Application**: Cash vs in-kind transfers