Problem Set 4

Econ 502: Advanced Microeconomics

Problem 1: Adverse Selection in the Used Laptop Market

A market for used laptops has two quality types:

- **High quality (H):** worth \$600 to sellers and \$1,000 to buyers
- **Low quality (L):** worth \$200 to sellers and \$400 to buyers

Let $\lambda \in [0, 1]$ denote the fraction of laptops that are high quality. Sellers know the quality of their own laptop; buyers cannot observe quality before purchase.

a) **Symmetric information.** Suppose buyers can perfectly observe quality. Describe the competitive equilibrium. Which types trade?

b) **Asymmetric information with $\lambda = 0.3$.** Buyers cannot observe quality and believe a fraction $\lambda = 0.3$ of laptops are high quality. What is a buyer's expected value of a laptop? Given this expected value, which sellers are willing to sell? What is the highest possible equilibrium price in this market?

c) **Asymmetric information with $\lambda = 0.6$.** Repeat part (b) with $\lambda = 0.6$. Does the market unravel in this case? Explain why the outcome differs from part (b).

Problem 2: Moral Hazard in Unemployment Insurance

A worker is laid off and enters a two-period model. In period 1, she is unemployed and receives UI benefit b. She chooses search effort $e \geq 0$ at personal cost $c(e)$. With probability e, she finds a job paying wage w in period 2; with probability $1 - e$, she remains unemployed and receives b again in period 2. The worker has utility $u(c)$ with $u' > 0$ and $u'' < 0$ (strictly risk-averse). The cost of search function $c(e)$ is increasing and convex, with $c(0) = 0$, $c' > 0$, and $c'' > 0$.

a) Write down the worker's expected utility as a function of e, b, and w. Derive the first-order condition for the optimal effort level e^* and give an economic interpretation.

b) Show that $\frac{de^*}{db} < 0$. Explain intuitively why higher benefits reduce search effort.

c) Now suppose $u(z) = \sqrt{z}$, $c(e) = \frac{1}{2}e^2$, and $w = 9$. Find $e^*(b)$ explicitly. At what benefit level does the worker exert zero effort?

Problem 3: Externalities and Regulatory Policy

A chemical plant produces q units of output per day and earns a profit:

$$\pi(q) = 80q - q^2$$

Its production generates pollution that imposes a total external cost on a neighboring farm of:

$$EC(q) = 10q$$

a) Find the chemical plant's privately optimal output q_{priv} and the associated profit. What is the total external cost imposed on the farm at this output level?

b) Find the socially efficient output q^* that maximizes total surplus (plant profit minus external cost). Calculate the deadweight loss from unregulated production.

c) Derive the Pigouvian tax t^* per unit of output that induces the plant to choose the efficient output. Verify that the tax achieves the social optimum.

d) **Coase Theorem.** Suppose bargaining between the plant and the farm is costless.

   i. **Farm holds the right** to zero pollution. Explain how the two parties negotiate. What output level emerges? What is the range of feasible payments from plant to farm?

   ii. **Plant holds the right** to pollute freely. Starting from the plant's privately optimal output, show that there are gains from trade from reducing output to q^*. Describe the negotiation and the range of feasible payments.

   iii. Compare the outcomes in (i) and (ii). What does the Coase theorem predict, and how does the assignment of property rights matter?

Problem 4: Public Goods and Free Riding

Two roommates (1 and 2) share an apartment and jointly consume a "household quality" public good G (WiFi, cleaning supplies, shared subscriptions) funded by their individual contributions $g_i \geq 0$, so $G = g_1 + g_2$. Roommate 1 has income $Y_1 = 120$ and roommate 2 has income $Y_2 = 180$. Each roommate's budget constraint is $x_i + g_i = Y_i$, and their utility is:

$$U_i(x_i, G) = x_i \cdot G$$

where $x_i = Y_i - g_i$ is private consumption.

Part I: Decentralized Equilibrium

a) Taking the other roommate's contribution as given, derive roommate i's best response function $g_i^*(g_j)$. How does the best response depend on g_j? What does this reveal about the free-rider problem?

b) Find the Nash equilibrium contributions (g_1^*, g_2^*), total public good G^*, private consumption x_i^*, and utility U_i^* for each roommate. Note any surprising feature of the equilibrium given the income difference.

Part II: Social Optimum

c) A social planner maximizes total welfare U_1+U_2 subject to the resource constraint $x_1+x_2+G = Y_1 + Y_2$. Find the efficient total public good G^{**}. (*Hint*: Let $X = x_1 + x_2$ and note that $U_1 + U_2 = X \cdot G$.)

d) If the planner allocates private consumption equally ($x_1^{**} = x_2^{**}$), find each roommate's contribution g_i^{**} and utility U_i^{**}. Compare G^* to G^{**}: by what fraction does decentralized provision fall short of the efficient level? Is each roommate better or worse off at the social optimum?

Part III: Policy

e) Suppose the government subsidizes contributions at rate s: each dollar contributed costs the roommate only $(1 - s)$ dollars (the government covers the rest). Find the subsidy rate s^* that induces the efficient total public good G^{**} in the Nash equilibrium.

Problem 5: Game Theory

Part I: Normal Form Games

Consider the following simultaneous-move game. The row player is A and the column player is B.

	L	**R**
U	(6, 2)	(0, 4)
D	(3, 3)	(5, 1)

a) Identify any strictly dominant or strictly dominated strategies. Eliminate any dominated strategies and explain your reasoning.

b) Find all pure-strategy Nash equilibria, or explain why none exist.

c) Find the unique mixed-strategy Nash equilibrium. Let p = probability A plays U and q = probability B plays L. Derive p^* and q^* and compute the expected payoff for each player.

Part II: Sequential Games

d) Now modify the game to be sequential: Player A moves first (choosing U or D), then Player B observes A's choice and responds. Draw the game tree. Using backward induction, find the subgame perfect equilibrium (SPE). What is the equilibrium outcome and what are the payoffs?

e) Compare the SPE outcome to the mixed-strategy equilibrium payoffs from part (c). Is moving first an advantage or a disadvantage for Player A in this game?
