Problem Set 3

Econ 502: Advanced Microeconomics

Market Structure and Welfare

Consider a market with inverse demand $P = 150 - Q$, where Q is total output. There are n identical firms, each with constant marginal cost $c = 30$ and no fixed costs.

Denote each firm's output as q_i for $i = 1, \ldots, n$, so that total output is $Q = \sum_{i=1}^{n} q_i$.

Part I: Benchmarks

a) **Perfect competition.** Suppose firms are price-takers ($P = MC$). Find the equilibrium price, total output, consumer surplus, and total surplus.

b) **Monopoly.** Suppose there is a single firm ($n = 1$). Derive the marginal revenue function. Find the profit-maximizing quantity, price, and profit. Calculate consumer surplus and the deadweight loss relative to perfect competition.

Part II: Price Competition

c) **Bertrand.** Suppose $n \geq 2$ firms compete by simultaneously setting prices. Consumers buy from the cheapest firm; if prices are tied, they split the market equally. What is the Nash equilibrium? Compare the outcome to your answer in part (a) and briefly explain the economic logic.

Part III: Quantity Competition

d) **Cournot duopoly.** Now suppose $n = 2$ firms compete by simultaneously choosing quantities, with total output $Q = q_1 + q_2$. Write down firm 1's profit function and derive its best response function $q_1^*(q_2)$. Find the Cournot-Nash equilibrium: each firm's output, the market price, and per-firm profit.

e) **Cournot with n firms.** Generalize part (d) to n identical firms. Show that in the symmetric Cournot-Nash equilibrium, per-firm output is given by:

$$q^* = \frac{120}{n+1}$$

f) **Convergence.** Using the result from part (e), fill in the following table and compute the markup μ, which is defined price relative to marginal cost:

$$\mu = \frac{P^*}{c}$$

n	Per-firm output (q^*)	Price (P^*)	Per-firm profit (π^*)	Markup (μ)
1				
2				
5				
10				
50				

What happens to the markup as n gets large?

Part IV: Welfare

g) **Deadweight loss.** Show that the deadweight loss from Cournot competition with n firms is given by:

$$DWL = \frac{7{,}200}{(n+1)^2}$$

How does the DWL change as n increases? Compare the monopoly DWL ($n = 1$) to the Cournot duopoly DWL ($n = 2$).