Problem Set 3: Solutions

Econ 502: Advanced Microeconomics

Part I: Benchmarks

Throughout: $P = 150 - Q$, $c = 30$, so $a - c = 120$.

Part (a): Perfect competition

Under perfect competition, $P = MC$:

$$150 - Q = 30 \implies Q_c = 120, \quad P_c = \$30$$

Consumer surplus is the triangle between the demand curve and the price (see figure below):

$$CS_c = \frac{1}{2}(150 - 30)(120) = \$7{,}200$$

Producer surplus is zero ($P = MC$ with constant marginal cost).

$$\boxed{P_c = \$30, \quad Q_c = 120, \quad CS_c = \$7{,}200, \quad TS_c = \$7{,}200}$$

Part (b): Monopoly

Total revenue: $TR = (150 - Q)Q = 150Q - Q^2$. Marginal revenue:

$$MR = 150 - 2Q$$

Setting $MR = MC$:

$$150 - 2Q = 30 \implies Q_m = 60, \quad P_m = 90$$

$$\pi_m = (90 - 30)(60) = \$3{,}600$$

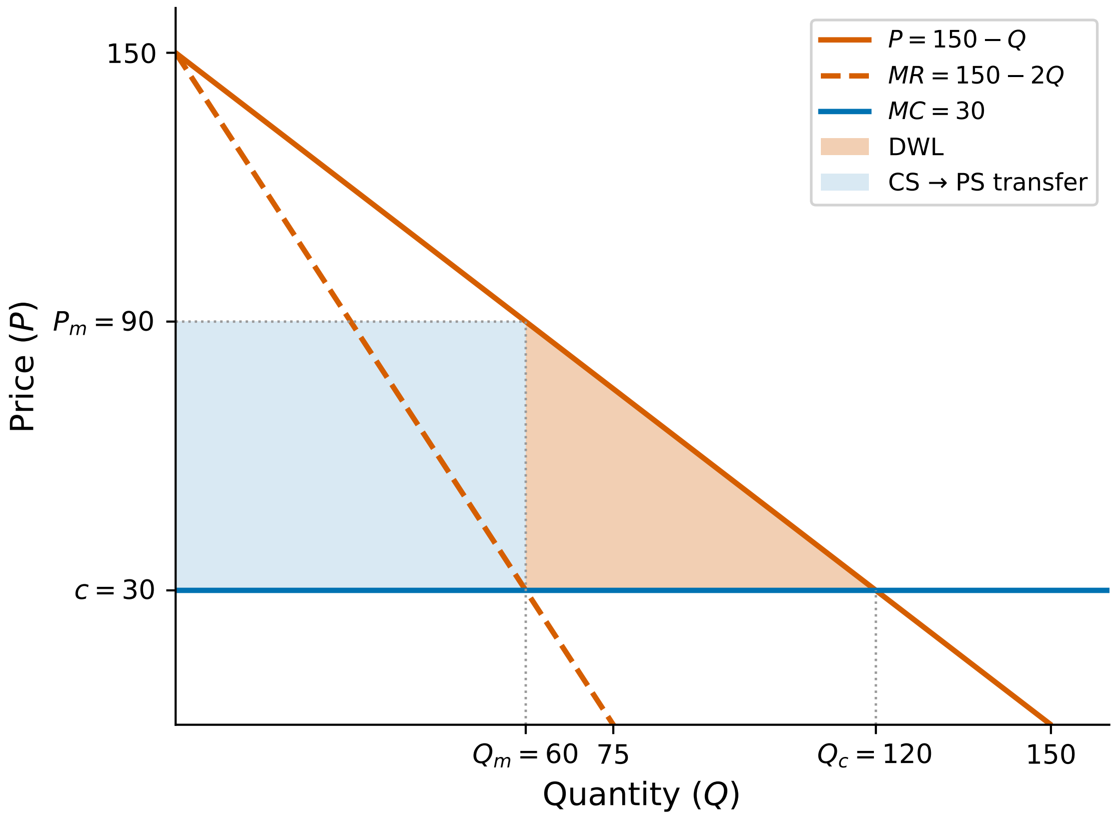
$$CS_m = \frac{1}{2}(150 - 90)(60) = \$1{,}800$$

$$TS_m = CS_m + \pi_m = 1{,}800 + 3{,}600 = \$5{,}400$$

$$DWL = TS_c - TS_m = 7{,}200 - 5{,}400 = \$1{,}800$$

This equals the triangle: $\frac{1}{2}(P_m - c)(Q_c - Q_m) = \frac{1}{2}(60)(60) = \$1{,}800$.

$$\boxed{Q_m = 60, \quad P_m = \$90, \quad \pi_m = \$3{,}600, \quad CS_m = \$1{,}800, \quad DWL = \$1{,}800}$$



Part II: Price Competition

Part (c): Bertrand

With $n \geq 2$ firms competing on price, the Nash equilibrium is:

$$\boxed{p_1 = p_2 = \cdots = p_n = c = 30}$$

This replicates the **perfectly competitive outcome**: $P = 30$, $Q = 120$, zero profits.

Why? Suppose all firms charge some $p > c$. Any single firm can capture the entire market by cutting its price to $p - \epsilon$, roughly multiplying its profit. This undercutting incentive continues until price is driven down to marginal cost. At $p = c$, no firm can profitably deviate: raising the price loses all customers, and lowering it means selling below cost.

This is the **Bertrand paradox**: just two firms are enough to eliminate all market power and reproduce the competitive outcome.

Part III: Quantity Competition

Part (d): Cournot duopoly ($n = 2$)

Firm 1's profit:

$$\pi_1 = (150 - q_1 - q_2)q_1 - 30q_1 = (120 - q_1 - q_2)q_1$$

First-order condition:

$$\frac{\partial \pi_1}{\partial q_1} = 120 - 2q_1 - q_2 = 0$$

Best response function:

$$\boxed{q_1^*(q_2) = 60 - \frac{q_2}{2}}$$

By symmetry, $q_2^*(q_1) = 60 - q_1/2$. In the symmetric equilibrium $q_1 = q_2 = q^*$:

$$q^* = 60 - \frac{q^*}{2} \quad \Longrightarrow \quad q^* = 40$$

$$Q^* = 80, \qquad P^* = 70, \qquad \pi^* = (70 - 30)(40) = \$1{,}600$$

$$\boxed{q^* = 40, \quad P^* = \$70, \quad \pi^* = \$1{,}600 \text{ per firm}}$$

Part (e): Cournot with n firms

Firm i maximizes:

$$\pi_i = \left(150 - q_i - \sum_{j \neq i} q_j\right) q_i - 30q_i$$

First-order condition:

$$120 - 2q_i - \sum_{j \neq i} q_j = 0$$

In a symmetric equilibrium, $q_j = q^*$ for all j, so $\sum_{j \neq i} q_j = (n-1)q^*$:

$$120 - 2q^* - (n-1)q^* = 0 \quad \Longrightarrow \quad 120 = (n+1)q^*$$

$$\boxed{q^* = \frac{120}{n+1}}$$

Part (f): Convergence

Remember $c = 30$ and:

$$q^* = \frac{120}{n+1}, \quad P^* = 150 - nq^*, \quad \pi^* = (P^* - c)q^*, \quad \mu = \frac{P^*}{c}$$

n	Per-firm output (q^*)	Price (P^*)	Per-firm profit (π^*)	Markup (μ)
1	60	\$90	\$3,600	3.00
2	40	\$70	\$1,600	2.33
5	20	\$50	\$400	1.67
10	10.9	\$40.9	\$119	1.36
50	2.35	\$32.9	\$6.8	1.09

As n gets large, price gets closer to marginal cost ($P^* \to 30$) and the markup μ approaches 1, replicating the competitive outcome.

Part IV: Welfare

Part (g): Deadweight loss

DWL is the triangle between the demand curve and the price, from Q^* to Q_c:

$$DWL = \frac{1}{2}(P^* - c)(Q_c - Q^*)$$

Note that

$$Q^* = nq^* = \frac{120n}{n+1}$$

Since $P^* = 150 - Q^*$, we can write:

$$P^* = 150 - \frac{120n}{n+1} = \frac{150 + 30n}{n+1}$$

Therefore:

$$DWL = \frac{1}{2}\left(\frac{150 + 30n}{n+1} - 30\right)\left(120 - \frac{120n}{n+1}\right) = \frac{7{,}200}{(n+1)^2}$$

Comparison:

- Monopoly ($n = 1$): $DWL = 7{,}200/4 = \$1{,}800$
- Cournot duopoly ($n = 2$): $DWL = 7{,}200/9 = \$800$

Going from monopoly to duopoly cuts the DWL by more than half. The DWL shrinks as $1/(n+1)^2$, so adding more firms reduces welfare loss rapidly. By $n = 5$, DWL is only \$200 (compared to \$1,800 under monopoly).