Problem Set 2

Econ 502: Advanced Microeconomics

Problem 1: Automation and Labor Demand

A manufacturing firm produces output using robots (R) and workers (L) with the CES production function:

$$Q = [0.6R^{0.5} + 0.4L^{0.5}]^2$$

The rental rate for robots is $r = 20$ per hour and the wage is $w = 25$ per hour.

a) What is the elasticity of substitution between robots and workers? What does this tell you about how easily the firm can replace workers with robots?

b) Solve for the cost-minimizing quantity of robots and workers to produce Q = 1000 units. Also, calculate the total cost of production at this input mix.

c) Now suppose technological progress reduces the robot rental rate to $r = 10$. How does the firm's use of robots and workers change? Calculate the new cost of production for 1000 units.

d) The government is considering a "robot tax" that would increase r back to 20 to protect jobs. Using your answer from (c), estimate how many jobs this would save per 1000 units of output. What's the trade-off?

Problem 2: Profit Maximization and Taxation

Keeping in mind firms' profit-maximizing behavior, would a lump-sum tax on profits affect the profit-maximizing quantity of output? What about a proportional tax on profits? What about a tax on each unit of output? How about a tax on labor input?

Problem 3: Trade Policy and the Portable Radio Market (Ex 12.8)

The domestic demand for portable radios is given by

$$Q = D(P) = 5{,}000 - 100P,$$

where price (P) is measured in dollars and quantity (Q) is the number of radios per year. The domestic supply curve for radios is given by

$$Q = S(P) = 150P.$$

a) What is the domestic equilibrium in the portable radio market?

b) Suppose portable radios can be imported at a world price of \$10 per radio. If trade were unencumbered, what would the new market equilibrium be? How many portable radios would be imported?

c) If domestic portable radio producers succeeded in having a \$5 tariff implemented, how would this change the market equilibrium? How much would be collected in tariff revenues? How much consumer surplus would be transferred to domestic producers? What would the deadweight loss from the tariff be?

d) How would your results from part (c) be changed if the government reached an agreement with foreign suppliers to "voluntarily" limit the portable radios they export to 1,250 per year? Explain how this differs from the case of a tariff.

Problem 4: General Equilibrium Exchange

Consider a simple exchange economy with two people (Ana and Ben) and two goods (X and Y). Total endowments: 100 units of X and 100 units of Y.

- **Ana:** Utility $U_A = X_A \cdot Y_A$, endowment $(X_A^0, Y_A^0) = (70, 30)$
- **Ben:** Utility $U_B = X_B \cdot Y_B$, endowment $(X_B^0, Y_B^0) = (30, 70)$

a) Calculate Ana's and Ben's marginal rate of substitution (MRS) at the initial endowment. Is this allocation Pareto efficient?

b) If they can trade at price ratio $P_X/P_Y = 1$, what will Ana demand? What will Ben demand? (Use their budget constraints based on the value of their endowments.)

c) Verify that the competitive equilibrium allocation is Pareto efficient by checking that $MRS_A = MRS_B$.

d) Describe the contract curve for this economy. What does the First Welfare Theorem tell us about competitive equilibrium?

Problem 5: Optimal Redistribution with Distortionary Taxation

A simple economy has two individuals:

- A **poor** individual with income y_P
- A **rich** individual with income $y_R > y_P$

The government chooses a **tax rate** $\tau \in [0, 1]$ applied only to the rich. All tax revenue is redistributed to the poor.

Taxation is distortionary: a fraction of revenue is lost due to inefficiency. Deadweight loss is given by:

$$\text{DWL} = \frac{1}{2}k\tau^2 y_R,$$

where $k > 0$ measures how costly taxation is.

The net transfer to the poor is:

$$T(\tau) = \tau y_R - \frac{1}{2}k\tau^2 y_R.$$

The poor's consumption is $c_P = y_P + T(\tau)$, and the rich's consumption is $c_R = y_R - \tau y_R$.

Social welfare is:

$$W(\tau) = \alpha c_P + (1-\alpha)c_R,$$

where $\alpha \in (0,1)$ represents society's inequality aversion.

a) Substitute the consumption expressions into the welfare function and simplify. Maximize welfare with respect to τ. Show that the optimal tax rate is:

$$\tau^* = \frac{2\alpha - 1}{k\alpha}.$$

b) Explain how the optimal tax rate τ^* changes with:

i. Inequality aversion α
ii. The distortion parameter k

What is the economic intuition behind each effect?
