Part 4: Game Theory Practice Problems

Econ 502: Advanced Microeconomics

1. Battle of the Sexes (Mixed Strategies)

A couple is deciding where to spend the evening. Player A prefers the **Opera**, Player B prefers the **Football** game, but both would rather be together than alone.

	B: Opera	**B: Football**
A: Opera	(3, 1)	(0, 0)
A: Football	(0, 0)	(1, 3)

a) Find the two pure-strategy Nash equilibria. Which player prefers each equilibrium? Why is this game called a "coordination game with conflict"?

b) Find the mixed-strategy Nash equilibrium. Let p be the probability A plays Opera and q be the probability B plays Opera. Derive p^* and q^*.

c) Compute each player's expected payoff in the mixed-strategy equilibrium. How does it compare to either of the pure-strategy equilibria? Why might players prefer to coordinate on a pure equilibrium even if the choice favors the other player?

d) Suppose A can commit (publicly and credibly) to going to the Opera before B chooses. What is the equilibrium outcome? What does this illustrate about the value of commitment?

2. Stag Hunt: Risk vs. Payoff Dominance

Two hunters can either cooperate to hunt a stag (large payoff but requires both) or hunt a hare alone (small payoff, guaranteed).

	B: Stag	**B: Hare**
A: Stag	(5, 5)	(0, 4)
A: Hare	(4, 0)	(4, 4)

a) Find all pure-strategy Nash equilibria. Show that one equilibrium is **payoff-dominant** (both players prefer it) and another is **risk-dominant** (each player's best response if they think the opponent is equally likely to choose either action).

b) Find the mixed-strategy Nash equilibrium. What is the probability each player puts on Stag?

c) Suppose A believes B will choose Stag with probability π. For what values of π does A play Stag? Use this to discuss why coordination failure (both choosing Hare) can be a stable outcome even though both players prefer (Stag, Stag).

d) The Stag Hunt is sometimes used as a metaphor for collective action problems (e.g., environmental treaties, vaccinations, financial crises). Pick one example and explain how the Stag Hunt structure applies.

3. Entry Deterrence with Capacity Commitment

An incumbent firm (I) faces a potential entrant (E). The game has two stages:

1. The incumbent chooses **capacity** $k \in \{$ Low, High $\}$ before E moves.
2. E observes k and chooses to **Enter** or **Stay Out**. If E enters, both firms compete in the market.

The payoffs (Incumbent, Entrant) for each combination are:

Capacity	E: Stay Out	E: Enter
Low	(12, 0)	(6, 3)
High	(8, 0)	(4, −2)

(Building capacity is a sunk cost for the incumbent - High capacity always yields lower profit for I than Low. But High capacity also commits the incumbent to flood the market post-entry, hurting the entrant enough that E stays out.)

a) Draw the game tree (extensive form).

b) Solve the game by backward induction. What is the subgame perfect equilibrium? What is the equilibrium outcome (capacity choice and entry decision)?

c) Suppose the incumbent could not commit to High capacity (i.e., capacity could be changed costlessly after E's decision). What would happen? Why does the ability to commit to High capacity benefit the incumbent?

d) Connect this game to the concept of a **non-credible threat**. If the incumbent simply *threatened* aggressive post-entry competition (without building capacity), would E believe it? Explain.

4. Ultimatum Bargaining

Two players bargain over \$10. Player 1 (the proposer) offers a split (s_1, s_2) where $s_1 + s_2 = 10$ and $s_2 \geq 0$. Player 2 (the responder) either accepts (each gets their proposed share) or rejects (both get 0). Assume s_2 can take any nonnegative real value (in dollars or cents).

a) Solve the game by backward induction. What does Player 2 do when offered $s_2 > 0$? What does Player 2 do when offered $s_2 = 0$? Assume that when indifferent, Player 2 accepts.

b) What is Player 1's optimal offer in the subgame perfect equilibrium? What are the equilibrium payoffs?

c) Empirically, in laboratory ultimatum games, responders frequently reject offers below about 30% of the pie, and proposers often offer close to 50–50. How does this evidence challenge the standard prediction? Suggest one modification of the standard model that could rationalize the observed behavior.

d) Now suppose the game has **two rounds**: if Player 2 rejects in round 1, Player 2 makes a counteroffer in round 2 over a pie that has shrunk to $\delta \cdot 10$ (where $\delta \in (0,1)$ is a discount factor reflecting the cost of delay). If Player 1 rejects the round-2 offer, both get 0. Solve by backward induction. How much does Player 1 offer in round 1? How does the equilibrium depend on δ?

5. Repeated Prisoner's Dilemma and Grim Trigger

Two firms play the following stage game in each period:

	B: Cooperate	**B: Defect**
A: Cooperate	(4, 4)	(0, 5)
A: Defect	(5, 0)	(1, 1)

a) Confirm that (Defect, Defect) is the unique Nash equilibrium of the one-shot game.

b) Suppose the game is repeated infinitely with discount factor $\delta \in (0,1)$. Each player considers the **grim trigger** strategy: "Cooperate in period 1, and continue to cooperate as long as the opponent has always cooperated; defect forever after the first deviation."

Compute the present discounted value of (i) always cooperating and (ii) deviating in period 1 (defect once and trigger permanent defection thereafter). Find the **critical discount factor** δ^* above which grim trigger sustains cooperation as a subgame perfect equilibrium.