Part 3: Market Failures Practice Problem Solutions

Econ 502: Advanced Microeconomics

Problem 1: Adverse Selection in Insurance

Part (a): Certainty equivalent and maximum premium

Without insurance, expected utility for a type-i consumer is

$$EU_i = (1-p_i)\sqrt{w} + p_i\sqrt{w-L} = (1-p_i)(10) + p_i(8) = 10 - 2p_i$$

The **certainty equivalent** CE_i is the certain wealth that yields the same utility as the gamble:

$$u(CE_i) = EU_i \implies \sqrt{CE_i} = 10 - 2p_i \implies CE_i = (10-2p_i)^2$$

Plugging in:

$$CE_L = (9.6)^2 = 92.16, \qquad CE_H = (8.8)^2 = 77.44$$

The consumer is indifferent between full insurance at premium $P_i^{\max}$ and going uninsured when $u(w - P_i^{\max}) = EU_i$, i.e., $w - P_i^{\max} = CE_i$:

$$\boxed{P_i^{\max} = w - CE_i}$$

$$P_L^{\max} = 100 - 92.16 = 7.84, \qquad P_H^{\max} = 100 - 77.44 = 22.56$$

Verification: $P_H^{\max} = 22.56 > 7.84 = P_L^{\max}$. ✓

Note that each willingness-to-pay exceeds the corresponding actuarially fair premium $p_i L$ ($p_L L = 7.20$, $p_H L = 21.60$). The gap is the **risk premium** - the extra amount the risk-averse consumer is willing to pay above the expected loss to avoid the gamble.

Part (b): Pooled premium at $\lambda = 0.5$

$$P_{\text{pool}} = (0.5 \cdot 0.6 + 0.5 \cdot 0.2) \cdot 36 = 0.4 \cdot 36 = 14.40$$

- **Low-risk:** $P_L^{\max} = 7.84 < 14.40$. They will **not** buy.
- **High-risk:** $P_H^{\max} = 22.56 > 14.40$. They **will** buy.

If only high-risk types buy, the insurer's expected payout per policy is $p_H \cdot L = 21.60$, but it collects only 14.40 per policy. The insurer **loses \$7.20 per policy** and does not break even.

Part (c): Range of λ for unraveling

The pooled premium as a function of the high-risk share is

$$P_{\text{pool}}(\lambda) = (0.4\lambda + 0.2) \cdot 36 = 14.4\lambda + 7.2$$

Low-risk types drop out when $P_{\text{pool}}(\lambda) > P_L^{\max}$:

$$14.4\lambda + 7.2 > 7.84 \implies \lambda > \frac{0.64}{14.4} = \frac{2}{45} \approx 0.044$$

$$\boxed{\text{Market unravels for } \lambda > 2/45 \approx 4.4\%}$$

Logic. At the pooled premium, low-risk types subsidize high-risk types. The maximum subsidy a low-risk type tolerates equals their risk premium ($P_L^{\max} - p_L L = 0.64$). As soon as the pool contains enough high-risk types to push the pooled premium above $P_L^{\max}$, low-risk consumers prefer to self-insure. Their exit raises the average risk in the pool, forcing the premium even higher - the classic adverse-selection death spiral. With only two types here, the dynamic stops once all low-risk are out and only high-risk remain (paying near $p_H L$).

Part (d): Compulsory insurance at $P_{\text{pool}} = 14.40$

Wealth under insurance is $w - P_{\text{pool}} = 85.60$ in both states, giving utility $\sqrt{85.60} \approx 9.252$.

Type	*EU* no insurance	u with mandate	Change
Low-risk	9.600	9.252	-0.348
High-risk	8.800	9.252	$+0.452$

- **Low-risk are worse off** (forced to subsidize the bad pool).
- **High-risk are better off** (get coverage at a price below their willingness to pay).

The insurer collects 14.40 per policy and pays out $0.5(0.6) + 0.5(0.2) = 0.4$ in probability $\times$ $L = 36$ on average $= 14.40$, so it **breaks even**.

Why the policy can still be desirable. A utilitarian planner adds the changes: net gain $= -0.348 + 0.452 = +0.104$ utils per person - total welfare rises. More fundamentally, the mandate solves the adverse-selection failure: high-risk types receive coverage they could otherwise obtain only at much higher prices, while low-risk types lose only their small risk-aversion benefit. There are also equity arguments: people generally do not choose their risk type, and pooling spreads the cost of bad luck. This logic underlies, e.g., the individual mandate in the Affordable Care Act.

Problem 2: Pigouvian Tax and Cap-and-Trade

Part (a): Unregulated equilibrium

Each plant ignores pollution damage and maximizes $y_i(P - c) = 2y_i$ subject to $y_i \leq 250$. The per-gallon profit is positive, so each plant produces at capacity:

$$y_1 = y_2 = 250$$

Total smog: $s_1 + s_2 = 250^2 + \frac{1}{2}(250^2) = 62{,}500 + 31{,}250 = 93{,}750$ cubic feet.

$$\boxed{\text{Total damage} = 0.01 \times 93{,}750 = \$937.50}$$

Part (b): Socially efficient outcome

The planner internalizes the damage. Each plant's social objective is

$$\max_{y_i} \, (P - c)y_i - 0.01 \cdot s_i(y_i)$$

Plant 1: max $2y_1 - 0.01y_1^2$. FOC: $2 = 0.02y_1 \implies y_1^* = 100$.

Plant 2: max $2y_2 - 0.005y_2^2$. FOC: $2 = 0.01y_2 \implies y_2^* = 200$.

Both interior (under capacity 250). The dirtier plant (Plant 1) is cut back more - exactly the asymmetric reallocation that command-and-control with a uniform cap would miss.

Total smog at the efficient outcome:

$$s_1^* + s_2^* = 100^2 + \tfrac{1}{2}(200^2) = 10{,}000 + 20{,}000 = 30{,}000 \text{ cubic feet}$$

$$\boxed{y_1^* = 100, \;\; y_2^* = 200, \;\; \text{total smog} = 30{,}000}$$

Part (c): Pigouvian tax

With tax $t = 0.01$ per cubic foot of smog, each plant now solves

$$\max_{y_i} \, (P - c)y_i - t \cdot s_i(y_i)$$

This is identical to the planner's problem in part (b), so each plant chooses the efficient output $y_1 = 100$, $y_2 = 200$. ✓

Total tax revenue: $t \times \text{total smog} = 0.01 \times 30{,}000 = \$300$.

The tax decentralizes the efficient outcome with minimal information: the regulator only needs to know the marginal damage of smog, not each plant's pollution function.

Part (d): Cap-and-trade

The regulator issues $\bar{S} = 30{,}000$ permits. Plant i holds initial allocation e_i (with $e_1 + e_2 = 30{,}000$) and faces market permit price τ. Each plant's profit is

$$\pi_i = (P - c)y_i - \tau(s_i(y_i) - e_i)$$

(the second term is the cost of using s_i permits on top of its endowment e_i - negative if the plant ends up a seller). The FOC is the same as under the tax: $P - c = \tau \cdot s_i'(y_i)$, so

$$2 = 2\tau y_1 \Longrightarrow y_1 = \frac{1}{\tau}, \qquad 2 = \tau y_2 \Longrightarrow y_2 = \frac{2}{\tau}$$

Market clearing. Total permit demand equals supply:

$$y_1^2 + \tfrac{1}{2}y_2^2 = \frac{1}{\tau^2} + \frac{2}{\tau^2} = \frac{3}{\tau^2} = 30{,}000$$

$$\tau^{*2} = \frac{1}{10{,}000} \quad \Longrightarrow \quad \tau^* = \$0.01$$

Plugging in: $y_1 = 100$, $y_2 = 200$ - same as the efficient allocation, and τ^* equals the Pigouvian tax rate.

(i) All permits to Plant 1 ($e_1 = 30{,}000$, $e_2 = 0$).

- Plant 1 uses $s_1 = 100^2 = 10{,}000$ permits and **sells the remaining 20{,}000 to Plant 2** at $\tau = 0.01$.
- Plant 1 profit: $\pi_1 = 100(2) - 0.01(10{,}000) + 0.01(30{,}000) = 200 - 100 + 300 = \$400$.
- Plant 2 profit: $\pi_2 = 200(2) - 0.01(20{,}000) + 0 = 400 - 200 = \$200$.

(ii) All permits to Plant 2 ($e_1 = 0$, $e_2 = 30{,}000$).

The first-order conditions and market-clearing equation are unchanged, so $\tau^* = 0.01$ and $y_1 = 100$, $y_2 = 200$ as before.

What changes is the trade direction and profits:

- Plant 2 keeps 20{,}000 permits for its own use and **sells 10{,}000 to Plant 1**.
- Plant 1 profit: $\pi_1 = 200 - 100 + 0 = \$100$.
- Plant 2 profit: $\pi_2 = 400 - 200 + 300 = \$500$.

Both plants produce the efficient quantities - only the distribution of profits shifts (totals are \$600 in both cases, redistributed by the permit revenue).

(iii) Connection to the Coase theorem.

With well-defined property rights (permits) and frictionless trade, the **efficient allocation arises regardless of who initially holds the rights**. The initial allocation only redistributes surplus; it has no effect on which plant pollutes how much. The regulator does not need to know plants' cost or pollution functions - the market reveals them through trading.

The price-based (Pigouvian tax) and quantity-based (cap-and-trade) instruments are dual: in this stylized environment, $\tau^* = t^*$ and the same outputs arise. Cap-and-trade has the additional virtue of fixing the *total* quantity of pollution (useful when the regulator cares about an absolute pollution ceiling), while the Pigouvian tax fixes the *price* of pollution (useful when there is uncertainty about quantities).

Problem 3: Coase Theorem

Part (a): Total surplus

	Doctor not SP	Doctor SP
Bakery noisy	260	$\boxed{300}$
Bakery quiet	280	250

The efficient combination is **bakery noisy + doctor soundproofed** (total surplus 300). Soundproofing eliminates a \$70 noise damage at a cost of only \$30, while letting the bakery keep its higher-profit noisy machinery.

Part (b): Doctor has the right to silence

Without bargaining. The bakery is forced to be quiet. The doctor compares 130 (no soundproofing) with 100 (soundproofing) and chooses **no soundproofing**. Outcome: (quiet, no SP), with payoffs $(\pi_B, \pi_D) = (150, 130)$ and total surplus 280.

With bargaining. The parties move to (noisy, SP), where the doctor agrees to soundproof and the bakery pays the doctor X for permission to be noisy. Joint surplus rises by 20.

- Bakery accepts if $200 - X \geq 150 \implies X \leq 50$
- Doctor accepts if $100 + X \geq 130 \implies X \geq 30$

$$\boxed{30 \leq X \leq 50}$$

Part (c): Bakery has the right to be noisy

Without bargaining. The bakery chooses to be noisy. The doctor compares 60 (no soundproofing) with 100 (soundproofing) and chooses to **soundproof**. Outcome: (noisy, SP), total surplus 300 - already efficient. **No bargaining is required.**

The doctor unilaterally adopts the efficient response because the cost of soundproofing (\$30) is less than the noise damage avoided (\$70).

Part (d): Comparison and the Coase theorem

In both regimes, the efficient outcome (noisy, SP) is reached with frictionless bargaining. **The assignment of property rights does not affect efficiency, only the distribution of payoffs:**

Regime	π_B	π_D
Doctor's right (bargained, $X = 30$)	170	130
Doctor's right (bargained, $X = 50$)	150	150
Bakery's right (no bargaining)	200	100

This is the **Coase theorem**: with well-defined property rights and zero transaction costs, private bargaining yields an efficient allocation regardless of the initial assignment of rights.

Why the assignment of rights still matters in practice:

- Real bargaining has **transaction costs** (lawyers, time, search) that may exceed the gains from trade.
- **Asymmetric information** about the parties' valuations can prevent agreement (each side may misrepresent reservation values).
- **Wealth and credit constraints**: if the loser cannot afford to pay the winner, the inefficient status quo persists.
- **Many parties** (multiple residents, multiple polluters) make Coasian bargaining infeasible due to free-riding and coordination costs.
- **Distributional / equity** considerations: even if efficient, society may prefer one assignment over another.

Problem 4: Public Good Provision

Part (a): Best response and Nash equilibrium

Person i chooses g_i to maximize

$$U_i = (Y_i - g_i)(g_i + g_{-i}) = (60 - g_i)(g_i + g_{-i})$$

The FOC is $-(g_i + g_{-i}) + (60 - g_i) = 0$, giving

$$\boxed{g_i^*(g_{-i}) = 30 - \frac{g_{-i}}{2}}$$

In a symmetric Nash equilibrium $g_1 = g_2 = g_3 = g$ and $g_{-i} = 2g$:

$$g = 30 - g \implies g_i^* = 15$$

$$\boxed{G^* = 45, \quad x_i^* = 45}$$

Part (b): Social optimum

Total welfare $\sum_i x_i G = XG$, with $X = \sum_i x_i$ and the resource constraint $X + G = 180$. Substituting,

$$\max_G \; G(180 - G) \implies G^{**} = 90$$

The equal allocation gives $X^{**} = 90$ and

$$\boxed{x_i^{**} = 30, \quad G^{**} = 90}$$

Part (c): Samuelson and the free-rider gap

For $U_i = x_i G$, $MRS_i = (\partial U_i/\partial G)/(\partial U_i/\partial x_i) = x_i/G$.

- **Nash:** $MRS_i = 45/45 = 1$ for each i. Each person privately equates MRS_i to the price of the public good, but $\sum_i MRS_i = 3$ - so collectively they massively under-provide.

- **Social optimum:** $MRS_i = 30/90 = 1/3$ for each i, and $\sum_i MRS_i = 1$. This is the **Samuelson condition** $\sum_i MRS_i = 1$ (the relative price of the public good in terms of the private good). ✓

$$\boxed{\frac{G^*}{G^{**}} = \frac{45}{90} = \frac{1}{2}}$$

Generalizing. With N symmetric contributors, the symmetric Nash satisfies $g_i^* = (60-(N-1)g_i^*)/2$, giving $g_i^* = 60/(N+1)$ and $G^* = 60N/(N+1)$. The social optimum gives $G^{**} = NY/2 = 30N$. So

$$\frac{G^*}{G^{**}} = \frac{2}{N+1}$$

As $N \to \infty$, this ratio $\to 0$. The free-rider problem becomes overwhelming as the number of contributors grows.

Part (d): Pigouvian-style subsidy

With subsidy rate s, the consumer's budget is $x_i = 60 - (1-s)g_i$ and

$$U_i = (60 - (1-s)g_i)(g_i + g_{-i})$$

FOC: $-(1-s)(g_i + g_{-i}) + (60 - (1-s)g_i) = 0$, giving $60 = (1-s)(2g_i + g_{-i})$.

Symmetric: $60 = (1-s) \cdot 4g \Longrightarrow g = 15/(1-s)$ and $G = 45/(1-s)$.

Setting $G = G^{**} = 90$:

$$\frac{45}{1-s} = 90 \quad \Longrightarrow \quad 1 - s = \frac{1}{2} \quad \Longrightarrow \quad \boxed{s^* = \frac{1}{2}}$$

The government covers half the cost of each contribution, exactly internalizing the externality each contributor imposes on the other two. (More generally, for N contributors, $s^* = (N-1)/(N+1)$.)