Part 3: Market Failures Practice Problems

Econ 502: Advanced Microeconomics

1. Adverse Selection in Insurance with Two Types

A population of consumers each has wealth $w = 100$. With probability p_i, they suffer a loss of $L = 36$. Consumers have utility $u(z) = \sqrt{z}$ and so are risk-averse. There are two risk types:

- **Low-risk** consumers have $p_L = 0.2$
- **High-risk** consumers have $p_H = 0.6$

A fraction λ of the population is high-risk. Each consumer knows their own type; the insurer cannot tell types apart. Insurance is "full insurance" at premium P (the insurer pays L if the loss occurs).

a) For each type $i \in \{L, H\}$, find the **certainty equivalent** CE_i of facing the loss without insurance. Use the certainty equivalent to compute the maximum premium $P_i^{\max}$ that makes the consumer just indifferent between buying full insurance and going uninsured. Verify that $P_H^{\max} > P_L^{\max}$.

b) Suppose the insurer charges the **actuarially fair pooled premium**:

$$P_{\text{pool}} = [\lambda p_H + (1 - \lambda)p_L] \cdot L$$

For $\lambda = 0.5$, compute P_{pool}. Which types are willing to buy at this premium? Will the insurer break even?

c) For what range of λ does the market unravel (low-risk types drop out, leaving only high-risk types)? Explain the economic logic in words.

d) Suppose the government mandates that everyone purchase insurance at the pooled premium P_{pool} with $\lambda = 0.5$. Compare each type's expected utility under the mandate with their expected utility without insurance. Who is better off? Who is worse off? Why might this still be a desirable policy from a social welfare perspective?

2. Pigouvian Tax and Cap-and-Trade

Two refineries produce fuel and emit smog as a byproduct. The market price of fuel is $P = \$3$ per gallon, and each plant uses $c = \$1$ in raw inputs per gallon. The two plants differ in pollution intensity:

$$s_1(y_1) = y_1^2, \qquad s_2(y_2) = \tfrac{1}{2}y_2^2$$

where y_i is gallons of fuel and s_i is cubic feet of smog. Each cubic foot of smog causes \$0.01 of environmental damage. Each plant's production capacity is $\bar{y} = 250$ gallons.

a) **Unregulated equilibrium.** Find each plant's privately optimal output (ignoring pollution damage). Compute total smog and total damage.

b) **Socially efficient outcome.** A planner internalizes the pollution damage. Find the efficient output of each plant. Compute total smog at the efficient outcome.

c) **Pigouvian tax.** Show that a per-unit tax $t = \$0.01$ on smog (paid by each plant on each cubic foot it emits) decentralizes the efficient outcome. What is total tax revenue?

d) **Cap-and-trade.** The regulator issues permits totaling the efficient quantity of smog from part (b), each allowing one cubic foot of pollution. Permits can be freely traded at a market price τ.

(i) Suppose **all permits are given to Plant 1**. Find the equilibrium permit price τ^* and the equilibrium outputs. How many permits does Plant 1 sell to Plant 2, and what are each plant's final profits?

(ii) Suppose instead **all permits are given to Plant 2**. Show that the equilibrium outputs and permit price are unchanged, but profits are reallocated.

(iii) Connect your findings to the **Coase theorem**.

3. Coase Theorem with Reciprocal Externalities

A bakery and a doctor share a building. The bakery's machinery generates noise. Profits depend on whether the bakery uses noisy or quiet machinery and whether the doctor's office is soundproofed:

	Doctor not soundproofed	Doctor soundproofed
Bakery noisy	$\pi_B = 200, \pi_D = 60$	$\pi_B = 200, \pi_D = 100$
Bakery quiet	$\pi_B = 150, \pi_D = 130$	$\pi_B = 150, \pi_D = 100$

Soundproofing costs the doctor \$30 (already netted out of π_D in the soundproofed column). Switching to quiet machinery costs the bakery \$50 in foregone profit.

a) Compute total surplus $\pi_B + \pi_D$ for each cell. Which combination is socially efficient?

b) **Right to silence.** Suppose the doctor has the legal right to a quiet office. Without bargaining, what does the bakery do, and does the doctor soundproof? With Coasian bargaining, what is the efficient outcome and what is the range of feasible payments from bakery to doctor?

c) **Right to be noisy.** Suppose the bakery has the legal right to make noise. Without bargaining, what does the doctor do? Is bargaining required to reach the efficient outcome?

d) Compare your answers in (b) and (c). What does this illustrate about the Coase theorem? Why might the assignment of property rights still matter in practice?

4. Public Good Provision with Three Contributors

Three neighbors share a small park. Each has income $Y_i = 60$ and chooses how much to contribute $g_i \geq 0$ to park maintenance. Total park quality is $G = g_1 + g_2 + g_3$. Each neighbor's utility is:

$$U_i(x_i, G) = x_i \cdot G$$

where $x_i = Y_i - g_i$ is private consumption.

a) Derive each neighbor's best response function $g_i^*(g_{-i})$, where $g_{-i} = \sum_{j \neq i} g_j$. By symmetry, find the Nash equilibrium contributions g_i^* and total provision G^*.

b) **Social optimum.** A planner chooses (x_1, x_2, x_3, G) to maximize $\sum_i U_i$ subject to $\sum_i x_i + G = \sum_i Y_i$. Find G^{**} and the equal-private-consumption allocation x_i^{**}. (Hint: with utility $x_i G$, total welfare is XG where $X = \sum x_i$.)

c) Verify that the Nash equilibrium satisfies $MRS_i = 1$ for each i but the social optimum satisfies $\sum_i MRS_i = 1$ (the **Samuelson condition**). Compute the ratio G^*/G^{**}. As the number of contributors N grows, what happens to the fraction of the efficient public good that is voluntarily provided?

d) Suppose each contributor receives a subsidy s on their contribution: a contribution of g_i costs them only $(1 - s)g_i$, with the government covering the rest. Find the subsidy rate s^* that decentralizes the efficient public good level G^{**}.