Part 2: Market Power Practice Problems

Econ 502: Advanced Microeconomics

1. Monopoly Pricing

A monopolist faces the inverse demand curve $P = 200 - 2Q$ and has constant marginal cost $MC = 40$ with no fixed costs.

a) Find the monopolist's profit-maximizing output Q_m^*, price P_m^*, and profit π. Show your derivation of marginal revenue.

b) What would be the competitive equilibrium price and quantity if this market were perfectly competitive? Calculate consumer surplus under perfect competition.

c) Under monopoly, calculate consumer surplus (CS_m), producer surplus (PS_m), and the deadweight loss (DWL). Show the DWL as both the difference in total surplus and as a triangle formula.

d) Calculate the Lerner Index at the monopoly equilibrium. Verify that it equals $1/|\varepsilon_D|$ using the price elasticity of demand at the monopoly point. Why does a profit-maximizing monopolist always operate on the elastic portion of its demand curve?

2. Third-Degree Price Discrimination

A museum can identify adult and student visitors and charge them different admission prices. The inverse demand curves are:

$$P_A = 100 - Q_A \qquad \text{(adults)}, \qquad P_S = 60 - Q_S \qquad \text{(students)}$$

The museum's marginal cost is $MC = 20$ and fixed costs are zero.

a) If the museum practices third-degree price discrimination, find the profit-maximizing price and quantity in each market and the total profit.

b) If the museum must charge a single uniform price to all visitors, find the profit-maximizing price, total quantity, and profit. (Hint: derive aggregate demand by horizontally summing the two demand curves.)

c) Under price discrimination, compute the Lerner Index and price elasticity of demand for each market. Verify that the inverse elasticity rule holds. Which group receives the higher markup, and why?

d) Compare total welfare (consumer surplus plus profit) under price discrimination and uniform pricing. Is price discrimination welfare-improving in this case? Explain the intuition.

3. Cournot Duopoly with Asymmetric Costs

Two firms compete in quantities with inverse demand $P = 120 - Q$, where $Q = q_1 + q_2$. Firm 1 has $MC_1 = 20$ and firm 2 has $MC_2 = 40$.

a) Derive each firm's best response function.

b) Find the Nash equilibrium quantities, the market price, and each firm's profit.

c) Compute the Lerner index for each firm. Which firm has more market power? Explain.

d) How does total output and welfare compare to the symmetric case where both firms have $MC = 30$? What drives the difference?

4. Bertrand Competition

Two firms sell a homogeneous good with inverse demand $P = 100 - Q$. Firm 1 has marginal cost $MC_1 = 10$ and Firm 2 has marginal cost $MC_2 = 20$. Each firm simultaneously sets its price; the lower-priced firm captures the entire market (with equal splitting at equal prices).

a) Show that $p_1 = p_2 = c$ (where c is the common marginal cost) is a Nash equilibrium when both firms have the same cost. Demonstrate that neither firm can profitably deviate.

b) Show that $p_1 = p_2 = p > c$ is **not** a Nash equilibrium when both firms have the same cost. What is the intuition?

c) With the asymmetric costs given above ($MC_1 = 10$, $MC_2 = 20$), find the equilibrium prices and profits. What determines the low-cost firm's equilibrium price?

d) The Bertrand model predicts that two firms are enough to achieve the competitive outcome (zero profit with identical costs). Describe two real-world departures from the Bertrand assumptions that allow firms to earn positive profits.

5. Differentiated Bertrand Competition

Two firms sell differentiated products with demand:

$$q_1 = 10 - 2p_1 + p_2, \qquad q_2 = 10 - 2p_2 + p_1$$

Both firms have marginal cost $c = 1$.

a) Derive each firm's best response function. Are prices strategic complements or substitutes? How do you know?

b) Find the Nash equilibrium prices, quantities, and profits.

c) Show that the equilibrium price exceeds marginal cost. How does product differentiation resolve the Bertrand paradox?

d) Suppose both firms collude and maximize joint profits, setting a common price p. Find the collusive price and each firm's profit. Compare to the Nash equilibrium. Can the collusion point be sustained as a Nash equilibrium in a one-shot game? Why or why not?

6. Stackelberg Competition

Two firms compete in quantities. Inverse demand is $P = 150 - Q$ where $Q = q_1 + q_2$, and both firms have marginal cost $c = 30$.

a) Derive each firm's best response function and find the Cournot Nash equilibrium quantities, price, and profits.

b) Now suppose firm 1 is the Stackelberg leader: it chooses q_1 first, and firm 2 observes this and then chooses q_2. Using backward induction, find the Stackelberg equilibrium quantities, price, and profits for each firm.

c) Compare the Cournot and Stackelberg outcomes from parts (a) and (b). Does the leader gain from moving first? What happens to the follower? Explain the "top dog" strategy.

d) Compute consumer surplus and total welfare under both Cournot and Stackelberg. Which outcome is better for consumers? For society?