Tax Incidence and Deadweight Loss

Econ 502: Advanced Microeconomics

Setup

Consider a competitive market in equilibrium. Let $Q^D(p)$ denote market demand and $Q^S(p)$ denote market supply. In equilibrium:

$$Q^D(p^*) = Q^S(p^*) = Q^*$$

Now impose a per-unit tax t on sellers. This creates a wedge between the price buyers pay (p^D) and the price sellers receive (p^S):

$$p^D = p^S + t$$

The new equilibrium requires:

$$Q^D(p^D) = Q^S(p^S)$$

Deriving the Tax Incidence Formula

Step 1: Differentiate the equilibrium condition

Starting from $Q^D(p^D) = Q^S(p^S)$ and $p^D = p^S + t$, totally differentiate both sides with respect to t:

$$\frac{dQ^D}{dp^D} \cdot \frac{dp^D}{dt} = \frac{dQ^S}{dp^S} \cdot \frac{dp^S}{dt}$$

Step 2: Use the tax wedge

Differentiating $p^D = p^S + t$ with respect to t:

$$\frac{dp^D}{dt} = \frac{dp^S}{dt} + 1$$

Substitute into Step 1:

$$\frac{dQ^D}{dp^D} \left(\frac{dp^S}{dt} + 1 \right) = \frac{dQ^S}{dp^S} \cdot \frac{dp^S}{dt}$$

Step 3: Solve for dp^S/dt

Expand:

$$\frac{dQ^D}{dp^D} \cdot \frac{dp^S}{dt} + \frac{dQ^D}{dp^D} = \frac{dQ^S}{dp^S} \cdot \frac{dp^S}{dt}$$

Rearrange:

$$\frac{dp^S}{dt} \left(\frac{dQ^S}{dp^S} - \frac{dQ^D}{dp^D} \right) = \frac{dQ^D}{dp^D}$$

$$\frac{dp^S}{dt} = \frac{dQ^D/dp^D}{dQ^S/dp^S - dQ^D/dp^D}$$

Step 4: Convert to elasticities

Define the elasticities at the initial equilibrium:

$$e_{D,p} = \frac{dQ^D}{dp} \cdot \frac{p^*}{Q^*} \quad \text{(negative)} \qquad e_{S,p} = \frac{dQ^S}{dp} \cdot \frac{p^*}{Q^*} \quad \text{(positive)}$$

So $dQ^D/dp = e_{D,p} \cdot Q^*/p^*$ and $dQ^S/dp = e_{S,p} \cdot Q^*/p^*$. Substitute:

$$\frac{dp^S}{dt} = \frac{e_{D,p} \cdot Q^*/p^*}{(e_{S,p} - e_{D,p}) \cdot Q^*/p^*} = \frac{e_{D,p}}{e_{S,p} - e_{D,p}}$$

Similarly, using $dp^D/dt = dp^S/dt + 1$:

$$\frac{dp^D}{dt} = \frac{e_{D,p}}{e_{S,p} - e_{D,p}} + 1 = \frac{e_{S,p}}{e_{S,p} - e_{D,p}}$$

Step 5: Take the ratio

$$\boxed{\frac{dp^D/dt}{dp^S/dt} = \frac{e_{S,p}}{e_{D,p}}}$$

Since $e_{D,p} < 0$, this ratio is negative. Taking absolute values, the side with the smaller elasticity (more inelastic) experiences the larger price change — and therefore bears more of the tax burden.

Deriving the Deadweight Loss Formula

Step 1: The Harberger triangle

The deadweight loss from the tax is the triangle between the supply and demand curves over the range of lost quantity:

$$DWL = \frac{1}{2} \cdot t \cdot |\Delta Q|$$

where $\Delta Q = Q_{\text{tax}} - Q^*$ is the reduction in equilibrium quantity caused by the tax.

Step 2: Find ΔQ in terms of elasticities

From the equilibrium condition $Q^D(p^D) = Q^S(p^S)$, the change in quantity can be expressed through either the demand or supply side. Using supply:

$$\Delta Q = \frac{dQ^S}{dp^S} \cdot \Delta p^S = \frac{dQ^S}{dp^S} \cdot \frac{dp^S}{dt} \cdot t$$

Substituting $dQ^S/dp = e_{S,p} \cdot Q^*/p^*$ and the result from the incidence derivation:

$$\Delta Q = \frac{e_{S,p} \cdot Q^*}{p^*} \cdot \frac{e_{D,p}}{e_{S,p} - e_{D,p}} \cdot t$$

Step 3: Substitute into the Harberger triangle

$$DWL = \frac{1}{2} \cdot t \cdot \left| \frac{e_{S,p} \cdot e_{D,p}}{e_{S,p} - e_{D,p}} \cdot \frac{Q^*}{p^*} \cdot t \right|$$

Since $e_{D,p} < 0$, the product $e_{S,p} \cdot e_{D,p} < 0$, and $e_{S,p} - e_{D,p} > 0$, so the fraction is negative. Taking the absolute value and pulling the negative through:

$$\boxed{DWL \approx -\frac{1}{2} \cdot \frac{e_{S,p} \cdot e_{D,p}}{e_{S,p} - e_{D,p}} \cdot \frac{t^2}{p^*} \cdot Q^*}$$

This expression is positive because the numerator $e_{S,p} \cdot e_{D,p} < 0$ and the leading negative sign flips it.

Key Takeaways

DWL grows with the square of the tax rate. The t^2 term means that doubling the tax rate quadruples the deadweight loss. This is the core intuition behind the Ramsey rule for optimal taxation: it is more efficient to levy small taxes across many goods than a large tax on one good.

Higher elasticities mean larger DWL. When buyers and sellers are more responsive to price changes, a tax causes a larger quantity distortion, generating more welfare loss.

Perfectly inelastic demand or supply implies zero DWL. If either $e_{D,p} = 0$ or $e_{S,p} = 0$, the numerator is zero and $DWL = 0$. Intuitively, if quantities don't change, there is no allocative distortion.