Worksheet: Single-Variable Unconstrained Optimization

ECON 441: Introduction to Mathematical Economics — Instructor: Div Bhagia

A point x_0 is a critical (or stationary) point of f if $f'(x_0) = 0$.

Find all the critical points of the following function:

$$f(x) = x^3 - 12x^2 + 36x + 8$$

Denote the two critical points by x_1^* and x_2^*, with $x_1^* < x_2^*$.

- What is the sign of $f'(x)$ for $x < x_1^*$ in the immediate neighborhood of x_1^*?
- What is the sign of $f'(x)$ for $x > x_1^*$ in the immediate neighborhood of x_1^*?
- What is the sign of $f'(x)$ for $x < x_2^*$ in the immediate neighborhood of x_2^*?
- What is the sign of $f'(x)$ for $x > x_2^*$ in the immediate neighborhood of x_2^*?

Using the first-derivative test, which critical point gives a relative maximum and which gives a relative minimum? What are the values of f at these points?

The second derivative of a function $f(x)$, written $f''(x)$, is the derivative of $f'(x)$. Find $f''(x)$ and evaluate it at x_1^* and x_2^*. What does the second-derivative test tell you about each critical point? Does it agree with your answer above?

Sketch the graph of $f(x) = x^3 - 12x^2 + 36x + 8$ for $0 \leq x \leq 8$, marking the two critical points.