Worksheet: Integration

ECON 441: Introduction to Mathematical Economics — Instructor: Div Bhagia

Rules of Integration

Integration reverses differentiation: if $F'(x) = f(x)$, then $\int f(x)\,dx = F(x) + c$.

- Power rule: $\int x^n\,dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$

  Example. $\int x^4\,dx = \frac{x^5}{5} + c$

- Exponential rule: $\int e^x\,dx = e^x + c$

- Log rule: $\int \frac{1}{x}\,dx = \ln x + c \quad (x > 0)$

- Sum and multiple: $\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx, \quad \int k f(x)\,dx = k\int f(x)\,dx$

  Example. $\int (6x^2 + 2)\,dx = 2x^3 + 2x + c$

- Definite integral: $\int_a^b f(x)\,dx = F(b) - F(a)$

  Example. $\int_0^2 3x^2\,dx = \left[x^3\right]_0^2 = 8 - 0 = 8$

Exercises

1. Find $\int (4x^3 - 2x + 1)\,dx$

2. Find $\int 3e^x\,dx$

3. Find $\int \frac{2}{x}\,dx$

4. Evaluate $\int_0^2 (x^2 + 1)\,dx$

Exercise. A firm invests at the rate $I(t) = 6t$ per year, and its capital stock at $t = 0$ is $K(0) = 50$.

(a) Find the capital stock $K(t)$.

(b) How much capital is accumulated between $t = 1$ and $t = 2$?