Inverse of a Matrix

ECON 441: Introduction to Mathematical Economics
Lecture 5

Div Bhagia

Inverse of a Matrix

For a **square** matrix A, its inverse A^{-1} is defined as:

$$AA^{-1} = A^{-1}A = I$$

Squareness is a **necessary** condition, not a **sufficient** condition

If a matrix's inverse exists, it is called a **nonsingular** matrix

Properties of Inverses

$$\left(A^{-1}\right)^{-1} = A$$

$$(AB)^{-1} = B^{-1}A^{-1}$$

$$(A^T)^{-1} = \left(A^{-1}\right)^T$$

Solution of Linear-Equation System

$$Ax = b$$

Pre-multiply both sides by A^{-1},

$$A^{-1}Ax = A^{-1}b \quad \Longrightarrow \quad x = A^{-1}b$$

If A is singular, a unique solution does not exist.

Conditions for Nonsingularity

Squareness is **necessary** but not **sufficient**

Sufficient condition for nonsingularity:

Rows (or equivalently) columns are linearly independent

Example.

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

A is singular, B is nonsingular.

Conditions for Nonsingularity

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad d = \begin{bmatrix} a \\ b \end{bmatrix}$$

We have a system of linear equations:

$$Ax = d$$

Then,

$$x_1 + 2x_2 = a$$
$$2x_1 + 4x_2 = b$$

Conditions for Nonsingularity

$$x_1 + 2x_2 = a$$
$$2x_1 + 4x_2 = b$$

For these equations to be consistent, we need $b = 2a$:

$$x_1 + 2x_2 = a$$

$$2x_1 + 4x_2 = 2a$$

Both are the same equation, infinite number of solutions.

Conditions for Nonsingularity

To summarize, for a matrix to be nonsingular (i.e. its inverse exists):

Necessary condition: **squareness**

Sufficient condition: **rows or (equivalently) columns are linearly independent**

Homework Questions

- Exercise 4.6: 6 (added to Practice Problems 4)
- Exercise 5.1: 1 – necessary and sufficient conditions (in Practice Problems 2)

Textbook reference: 4.6, 5.1