Functions, Summation Notation, and Logical Conditions

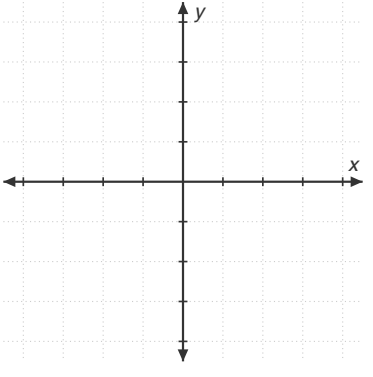
ECON 441: Introduction to Mathematical Economics
Lecture 2

Div Bhagia

Relations

Relation: subset of the Cartesian product

Example. $\{(x, y) | y \leq x\}$



Functions

Function: a relation where for each x there is a unique y

$$f : X \to Y, \quad y = f(x)$$

Examples. $y = x, y = x^2, y = 2x + 3$

X : domain, Y : codomain, $f(X)$: range

Most functions we will encounter, $f : \mathbb{R}^k \to \mathbb{R}$

Functions

Let's say,

$$f : X \to \mathbb{R}, \quad y = 3x - 5$$

where $X = \{2, 3, 4\}$. What is the range?

Cost Function

Consider the total cost C of producing hats Q,

$$C = f(Q)$$

- Should I be able to produce 10 hats and 5 hats at the same cost?
- Possible to produce 5 hats for $20 and $25?

By the way

Consider the total cost C of producing hats Q,

$$C = f(Q) = 2Q + 5$$

What is the cost of producing 1 hat? What is the cost of producing 2 hats? How many hats can I produce for $25?

Types of Functions

- Constant: $y = f(x) = 5$
- Polynomial of degree n: $y = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$
  - $n = 0$, constant
  - $n = 1$, linear
  - $n = 2$, quadratic
  - $n = 3$, cubic
  - ...
- Rational function: ratio of two polynomial functions:

$$y = \frac{a}{x}$$

Function of More than One Variables

Functions can be of two variables:

$$z = g(x, y)$$

Or three, or four,..., or n

Monotonic functions

Strictly increasing function:

$$x_1 > x_2 \rightarrow f(x_1) > f(x_2)$$

Strictly decreasing function:

$$x_1 > x_2 \rightarrow f(x_1) < f(x_2)$$

Increasing function:

$$x_1 > x_2 \rightarrow f(x_1) \geq f(x_2)$$

Decreasing function:

$$x_1 > x_2 \rightarrow f(x_1) \leq f(x_2)$$

Inverse of a function

Function $y = f(x)$ has an inverse if it is a one-to-one mapping, i.e. each value of y is associated with a unique value of x. Inverse function

$$x = f^{-1}(y)$$

returns the value corresponding value of x for each y. One-to-one mapping unique to strictly monotonic functions

Inverse of a function

Example: Find the inverse of $y = f(x) = 3x - 2$.

Summation Notation

$$\sum_{i=1}^{N} x_i = x_1 + x_2 + \ldots + x_N$$

Example: $x = \{2, 9, 6, 8, 11, 14\}$. What is $\sum_{i=1}^{4} x_i$?

Summation Notation

Another way of using a summation sign is to write

$$\sum_{x \in A} x$$

which refers to summing up all elements in A. To sum up x for all possible values x, we can simply write

$$\sum_{x} x$$

Things you CAN do

1. Pull constants out of or into the summation sign.

$$\sum_{i=1}^{N} bx_i = b \sum_{i=1}^{N} x_i$$

Things you CAN do

2. Split apart (or combine) sums (addition) or differences (subtraction)

$$\sum_{i=1}^{N}(bx_i + cy_i) = b\sum_{i=1}^{N} x_i + c\sum_{i=1}^{N} y_i$$

Things you CAN do

3. Multiply through constants by the number of terms in the summation

$$\sum_{i=1}^{N}(a + bx_i) = aN + b\sum_{i=1}^{N} x_i$$

Things you CANNOT do

1. Split apart (or combine) products (multiplication) or quotients (division).

$$\sum_{i=1}^{N} x_i y_i \neq \sum_{i=1}^{N} x_i \times \sum_{i=1}^{N} y_i$$

Things you CANNOT do

2. Move the exponent out of or into the summation.

$$\sum_{i=1}^{N} x_i^a \neq \left(\sum_{i=1}^{N} x_i \right)^a$$

Necessary vs. Sufficient Conditions

q is a necessary condition for p if:

$$p \implies q$$

p: I ate tofu for dinner

q: My dinner had protein

Necessary vs. Sufficient Conditions

q is a sufficient condition for p if:

$$p \Longleftarrow q$$

p: A number is even

q: A number is divisible by 4

Necessary vs. Sufficient Conditions

q is both necessary and sufficient for p

$$p \iff q$$

p: A number is even

q: A number is divisible by 2

Necessary vs. Sufficient Conditions

p: It is a holiday

q: It is Thanksgiving

Necessary vs. Sufficient Conditions

p: The car is out of gas

q: The car isn't starting

Necessary vs. Sufficient Conditions

p: A geometric figure has four sides

q: It is a rectangle

Homework Questions

- Exercise 2.3: 1, 2
- Exercise 2.4: 5, 7, 8
- Exercise 2.5: 1 (For each part, find the inverse of the function too.)
- Exercise 4.2: 6, 8
- Exercise 5.1: 1