

Integration

ECON 441: Introduction to Mathematical Economics Lecture 13

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Inverse of Differentiation

Path of population over time:

$$P(t) = 2t^{0.5}$$

Rate of change of population:

$$P'(t) = \frac{dP}{dt} = t^{-0.5}$$

But what if instead we were given $P'(t)$ and were tasked with finding $P(t)$?

Inverse of Differentiation

Note that

$$P'(t) = \frac{dP}{dt} = t^{-0.5}$$

is the derivative of $P(t) = 2t^{0.5}$, but also of $P(t) = 2t^{0.5} + 30$.

Generally, at best, we can find the following from just $P'(t)$:

$$P(t) = 2t^{0.5} + c$$

However, if we have an initial condition such as $P(0) = 50$, we can also find c .

Example: From Marginal Cost to Total Cost

A firm's marginal cost is

$$MC(Q) = 6Q + 2$$

and its fixed cost is 8, so $C(0) = 8$.

Marginal cost is the derivative of total cost: $MC(Q) = C'(Q)$.

Can we recover the total cost function $C(Q)$?

Integration

- Integration is the reverse of differentiation
- If $f(x)$ is the derivative of $F(x)$, we can **integrate** $f(x)$ to find $F(x)$:

$$\frac{d}{dx}F(x) = f(x) \Rightarrow \int f(x) dx = F(x) + c$$

- Rules of integration follow from rules of differentiation

Rules of Integration

Power rule

$$\int x^n dx = \frac{1}{n+1} \cdot x^{n+1} + c \quad (n \neq -1)$$

Examples:

$$\int x^3 dx, \quad \int x dx, \quad \int 1 dx$$

Rules of Integration

Exponential rule

$$\int e^x dx = e^x + c$$

Log rule

$$\int \frac{1}{x} dx = \ln x + c \quad (x > 0)$$

Rules of Integration

Integral of a sum

$$\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx$$

Integral of a multiple

$$\int kf(x) dx = k \int f(x) dx$$

Example:

$$\int (x^2 + 3x + 1) dx =$$

Definite Integrals

Definite integral:

$$\int_a^b f(x) dx = \left[F(x) \right]_a^b = F(b) - F(a)$$

Example:

$$\int_1^3 2x^2 dx =$$

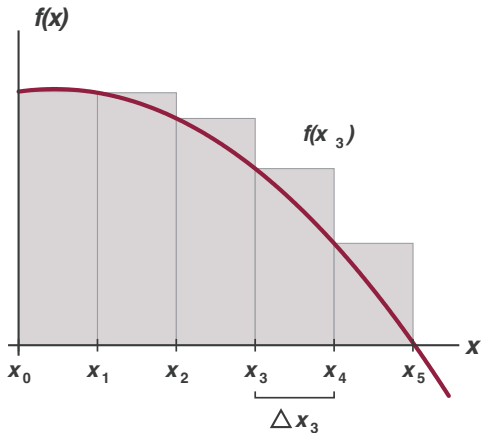
Example: Investment and the Capital Stock

A firm's capital stock $K(t)$ grows through investment: $I(t) = \frac{dK}{dt}$.

Suppose $I(t) = 2t + 5$ and the initial capital stock is $K(0) = 100$.

- Find the path of the capital stock $K(t)$.
- How much capital is accumulated between $t = 2$ and $t = 4$?

Area Under the Curve



$$\text{Area} \approx \sum_{i=1}^n f(x_i) \Delta x_i$$

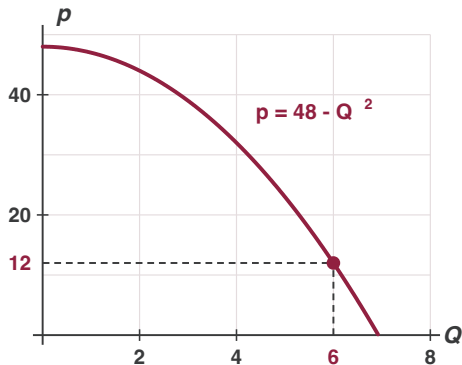
Area Under the Curve



$$\begin{aligned}\text{Area} &= \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) \Delta x_i \\ &= \int_{x_1}^{x_n} f(x) dx\end{aligned}$$

Example: Consumer Surplus

Demand: $p = 48 - Q^2$. Market price: $p = 12$. Find the consumer surplus.



Homework Questions

- Exercise 14.2: 1 (a), (c), (d)
- Exercise 14.3: 1 (a), (e), 2 (a), (d), 5

Textbook reference: 14.1-14.3

Midterm 2 is next week: review on Monday, exam on Wednesday

A sample exam and a help sheet are posted on the Exams page