

Implicit Functions

ECON 441: Introduction to Mathematical Economics
Lecture 12

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Implicit Functions

Explicit function:

$$y = f(x_1, x_2, \dots, x_n)$$

Implicit function:

$$F(y, x_1, x_2, \dots, x_n) = 0$$

Example

Implicit function:

$$F(x, y) = y - 3x^2 = 0$$

Corresponding explicit function:

$$y = f(x) = 3x^2$$

However, not all implicit functions have a corresponding explicit function. E.g. $F(x, y) = x^2 + y^2 - 9 = 0$

Implicit Function Theorem

Given,

$$F(x, y) = 0$$

If the following conditions are met:

- F_y and F_x are continuous, and
- At some point (a, b) , F_y is non-zero

Then in a neighborhood around (a, b) , an implicit function exists. Moreover, this function is continuous and has continuous partial derivatives.

Derivatives of Implicit Functions

Totally differentiating F , we have $dF = 0$, or

$$F_y dy + F_1 dx_1 + \cdots + F_n dx_n = 0$$

Suppose that only y and x_1 are allowed to vary:

$$\frac{\partial y}{\partial x_1} = -\frac{F_1}{F_y}$$

In the simple case where the given equation is $F(y, x) = 0$, the rule gives

$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$

Example: Market Equilibrium

Demand: $Q^d = 100 - 2P + 0.02Y$, where Y is income

Supply: $Q^s = 20 + 3P - c$, where c is an input cost

In equilibrium $Q^d = Q^s$, that is,

$$F(P, Y, c) = 80 - 5P + 0.02Y + c = 0$$

This defines the equilibrium price P as an implicit function of Y and c .
Find $\partial P / \partial Y$ and $\partial P / \partial c$.

Check your answer by solving for P explicitly.

Example: When You Cannot Solve for y

Given the following function, let's find $\partial y / \partial x$ and $\partial y / \partial z$.

$$F(x, y, z) = x^3 z^2 + y^3 + 4xyz = 0$$

Another Example

Estimate the following model for demand for fast food:

$$\text{orders} = \beta_0 + \beta_1 \text{ price} + \beta_2 \text{ quality} + \varepsilon$$

What is the interpretation of β_1 ?

Another Example (cont.)

What if instead we estimate:

$$\ln(\text{orders}) = \beta_0 + \beta_1 \ln(\text{price}) + \beta_2 \text{quality} + \varepsilon$$

What is the interpretation of β_1 ?

Another Related Example

Production function:

$$Y = AL^{\alpha}K^{\beta}$$

To estimate the elasticities from data:

$$\ln Y = \ln A + \alpha \ln L + \beta \ln K + \varepsilon$$

Elasticity in Logs

Recall that elasticity is a ratio of percentage changes:

$$\varepsilon = \frac{dQ/Q}{dP/P}$$

Since $d \ln Q = \frac{dQ}{Q}$ and $d \ln P = \frac{dP}{P}$,

$$\varepsilon = \frac{d \ln Q}{d \ln P}$$

So if $\ln Q = a - \alpha \ln P$, then $\varepsilon = -\alpha$

Find the Derivative by Taking the Log

Demand: $Q(p) = \frac{c}{p^\alpha}$

Homework Questions

- Exercise 8.5: 1, 2 (d), 3 (a)

Textbook reference: 8.5

Midterm 2 is on Wednesday, October 21: a sample exam and a help sheet are posted on the Exams page