Numbers and Sets

ECON 441: Introduction to Mathematical Economics
Lecture 1

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Introductions

- Your preferred name and pronouns
- Why are you here?
  - Undergrads: why you signed up for this class
  - Grad students: why you're doing the MA
- The last math class you took, and roughly when
- Your comfort food

Real-Number System

- **Integers**:

$$\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$$

- **Fractions**:

$$\frac{1}{2}, \frac{3}{5}, -\frac{2}{3}$$

- **Rational numbers**: ratio of integers
- **Are fractions rational numbers? What about integers?**

Real-Number System

- **Rational numbers**: ratio of integers
  - "terminating or repeating decimal"
  - Example. $\frac{1}{3} = 0.333$, $\frac{1}{4} = 0.25$

- **Irrational numbers**: cannot be expressed as a ratio of two integers
  - "nonterminating or nonrepeating decimal"
  - Example. $\sqrt{2} = 1.4142$, $\pi = 3.1415$

- **Real numbers ($\mathbb{R}$)**: rational and irrational

Sets

- A **set** is a collection of distinct objects.

$$A = \{brownies, icecream, pizza, ramen\}$$

- $pizza \in A$, $\in$ stands for 'is in'

- Other ways of writing a set

$$B = \{x|x \text{ is a positive integer}\}$$

$$C = \{x|1 < x < 5\}$$

Set Relations

- Equivalence (=)

$$A = \{brownies, icecream, pizza, ramen\}$$

$$B = \{pizza, ramen, icecream, brownies\}$$

$$A = B$$

- Subset ($\subset$)

$$C = \{pizza, ramen\}$$

$C \neq A$ but $C \subset A$. Note: $A \supset C$ is equivalent. **Is $A \subset B$?**

Set Relations

- Disjoint sets

$$A = \{brownies, icecream, pizza, ramen\}$$

$$D = \{salad, fruits\}$$

- Neither but still related

$$A = \{brownies, icecream, pizza, ramen\}$$

$$E = \{salad, fruits, icecream\}$$

Set Relations

- $\varnothing$: empty or null set

- What are all possible subsets of $S = \{a, b, c\}$?
  - $\varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}$
  - Always 2^n subsets. Here $n = 3$, so 8 subsets.

Set Operations

- **Union:** $A \cup B$, elements in either A **or** B
- **Intersection:** $A \cap B$, elements in both A **and** B
- Example:

$$A = \{brownies, icecream, pizza, ramen\}$$

$$B = \{salad, fruits, icecream\}$$

  - $A \cup B =$
  - $A \cap B =$

Set Operations

- **Union:** $A \cup B$, elements in either A or B
- **Intersection:** $A \cap B$, elements in both A and B
- What about

$$A = \{brownies, icecream, pizza, ramen\}$$

$$B = \{salad, fruits\}$$

  - $A \cup B =$
  - $A \cap B =$

Set Operations

- **Complement of A**: $\tilde{A}$ or A^c, 'not A'
- Universal set U (context specific) then:

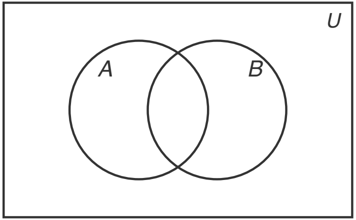
$$A^c = \{x | x \in U \text{ and } x \notin A\}$$

Example. $U = \{1, e, f, 2\}, A = \{1, 2\}$, then $A^c = \{e, f\}$.

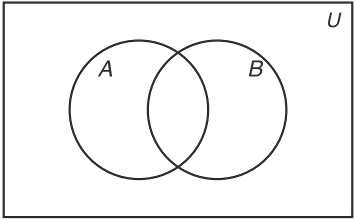
Set Operations: Venn Diagrams

Shade the following regions:

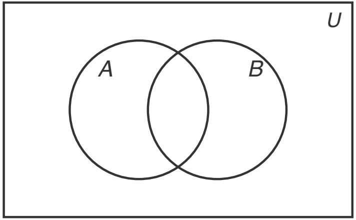
$A \cup B$



$A \cap B$



A^c



Laws of Set Operations

- Commutative law

$$A \cup B = B \cup A \qquad A \cap B = B \cap A$$

- Distributive law

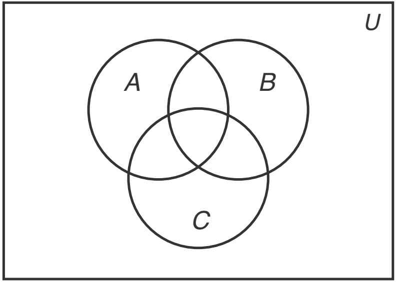
$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

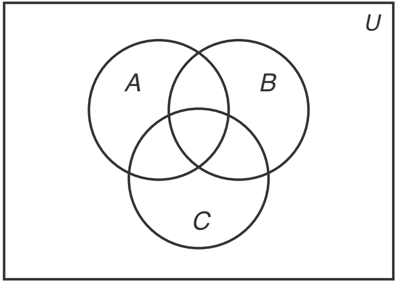
Distributive law

Verify by shading: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

A



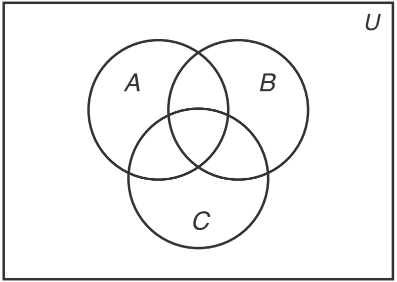
$B \cap C$



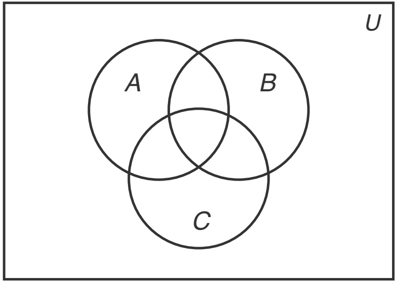
Distributive law

Now the right-hand side, $(A \cup B) \cap (A \cup C)$:

$A \cup B$



$A \cup C$



Ordered Sets

- We said order does not matter for sets
- But we can have ordered sets where

$$(a, b) \neq (b, a) \text{ unless } a = b$$

- Ordered pairs, triples,...
  - Example. $(age, weight)$, $(22, 120)$ different from $(120, 22)$

Cartesian Product

The **Cartesian product** of two sets A and B is the set of all ordered pairs whose first element comes from A and second from B:

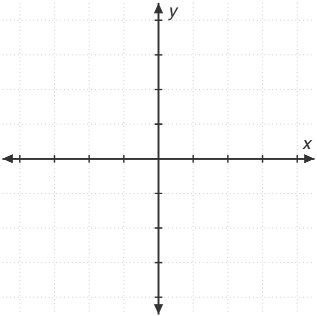
$$A \times B = \{(a, b) \mid a \in A,\ b \in B\}$$

Example. $A = \{1, 2\}$, $B = \{3, 4\}$

$$A \times B = \{(1, 3), (1, 4), (2, 3), (2, 4)\}$$

Cartesian Plane

$$\mathbb{R}^2 = \{(x, y) | x \in \mathbb{R}, y \in \mathbb{R}\}$$



Can have $\mathbb{R}^3, \mathbb{R}^4, \ldots, \mathbb{R}^n$

By the way

What is $x \times x$?

What is $x^2 \times x$?

What is $x^2 \times x^2$?

More generally, $x^n \times x^m = x^{m+n}$.