

Midterm 2 Review

ECON 441: Introduction to Mathematical Economics

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What's on the exam

- Wednesday, October 21, in class. 60 minutes, 20 points.
- Module III: Calculus (Lectures 8–12; integration is not covered)
- A sample exam with solutions and the help sheet provided with the exam are on the Exams page
- Closed book: no phones or computers
- For your review: lecture slides and worksheets, Practice Problems 8–12 (also collected as one PDF for the module), the module notes, and additional problems in the textbook

The Derivative

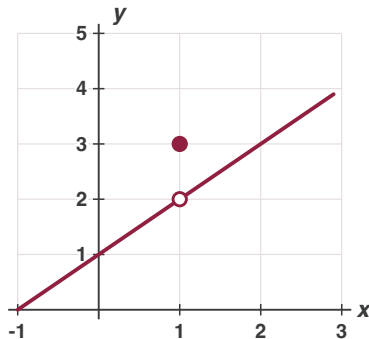
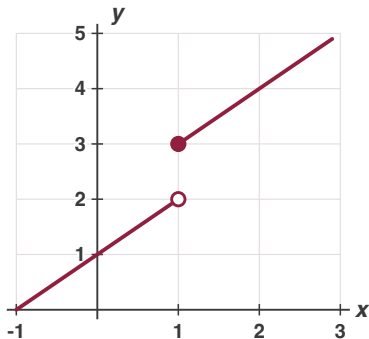
Average rate of change, then let the change shrink:

$$\frac{dy}{dx} = f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Use the limit definition to find $f'(x)$ for $y = f(x) = x^2 + 3x$.

Limits and Continuity

$\lim_{x \rightarrow a} f(x)$ exists only if the left-side and right-side limits agree; f is **continuous** at a if the limit exists and equals $f(a)$.



Continuity and Differentiability

Continuity is **necessary** for differentiability, but not **sufficient**.

$$g(x) = \begin{cases} 2x & \text{if } x < 1 \\ x + 1 & \text{if } x \geq 1 \end{cases}$$

Plot g . Is it continuous at $x = 1$? Is it differentiable there?

The Rules

Product, Quotient, and Chain Rule:

$$\frac{d}{dx}[fg] = fg' + f'g \qquad \frac{d}{dx} \frac{f}{g} = \frac{f'g - fg'}{g^2} \qquad \frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$$

Find dy/dx :

- $y = (x^2 + 1)(3x - 2)$
- $y = \frac{x}{x + 2}$
- $y = (2x + 1)^3$

Exponential and Log Functions

$$\frac{d}{dx}e^x = e^x \qquad \frac{d}{dx}\ln x = \frac{1}{x}$$

For $e^{f(x)}$ and $\ln f(x)$, combine these with the chain rule. Find dy/dx :

- $y = 3e^{2x}$
- $y = x^2 \ln x$
- $y = \ln(x^2 + 4)$

Elasticity and Logs

$$\varepsilon = \frac{dy}{dx} \cdot \frac{x}{y} = \frac{d \ln y}{d \ln x}$$

Demand is $Q = \frac{50}{p^2}$. What is the price elasticity of demand? Is demand elastic or inelastic?

Partial Derivatives

Differentiate with respect to one variable, holding the others constant.

For $f(x, y) = x^2y + y^3$, find f_x , f_y , and the gradient ∇f . Is f_x increasing in y ?

Production Function

$$Q = K^{0.3}L^{0.7}$$

Find the marginal product of capital Q_K and of labor Q_L .

Is the marginal product of labor decreasing in L ?

Total Differential and Total Derivative

For $y = f(x_1, x_2, \dots, x_n)$, the **total differential** is

$$dy = f_1 \cdot dx_1 + f_2 \cdot dx_2 + \dots + f_n \cdot dx_n$$

If the other variables depend on x_1 , the **total derivative** with respect to x_1 is

$$\frac{dy}{dx_1} = f_1 + f_2 \cdot \frac{dx_2}{dx_1} + \dots + f_n \cdot \frac{dx_n}{dx_1}$$

Example

$$z = x^2 + xy, \quad \text{where } x = t \text{ and } y = t^2$$

Find the total differential dz and the total derivative dz/dt .

Implicit-Function Rule

For $F(x, y) = 0$, totally differentiate: $F_x \cdot dx + F_y \cdot dy = 0$, so

$$\frac{dy}{dx} = -\frac{F_x}{F_y}$$

Market Equilibrium

Demand: $Q^d = Y - P^2$, with Y income

Supply: $Q^s = P + \ln P$

The equilibrium condition cannot be solved for P explicitly. Write it as $F(P, Y) = 0$ and find $\partial P / \partial Y$. Does higher income raise the price?