

Practice Problems: Calculus

ECON 441: Introduction to Mathematical Economics

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Exercise 6.2

2. Given the function $y = 5x^2 - 4x$:

- (a) Find the difference quotient as a function of x and Δx .
- (b) Find the derivative dy/dx , using the limit definition.
- (c) Find $f'(2)$ and $f'(3)$.

3. Given the function $y = 5x - 2$:

- (a) Find the difference quotient $\Delta y/\Delta x$. What type of function is it?
- (b) Since Δx does not appear in $\Delta y/\Delta x$ in part (a), does it make any difference to the value of $\Delta y/\Delta x$ whether Δx is large or small? Consequently, what is the limit of the difference quotient as Δx approaches zero?

Exercise 7.1

3. Find $f'(1)$ and $f'(2)$ for the following functions:

(a) $y = f(x) = 18x$

(c) $f(x) = -5x^{-2}$

(e) $f(w) = 6w^{1/3}$

(b) $y = f(x) = cx^3$

(d) $f(x) = \frac{3}{4}x^{4/3}$

(f) $f(w) = -3w^{-1/6}$

Exercise 7.2

3. Differentiate the following by using the product rule:

(d) $(ax - b)(cx^2)$

(e) $(2 - 3x)(1 + x)(x + 2)$

7. Find the derivatives of:

(a) $(x^2 + 3)/x$

(c) $6x/(x + 5)$

(b) $(x + 9)/x$

(d) $(ax^2 + b)/(cx + d)$

8. Given the function $f(x) = ax + b$, find the derivatives of:

(a) $f(x)$

(b) $xf(x)$

(c) $1/f(x)$

(d) $f(x)/x$

Exercise 7.3

1. Given $y = u^3 + 2u$, where $u = 5 - x^2$, find dy/dx by the chain rule.

2. Given $w = ay^2$ and $y = bx^2 + cx$, find dw/dx by the chain rule.

3. Use the chain rule to find dy/dx for the following:

(a) $y = (3x^2 - 13)^3$

(b) $y = (7x^3 - 5)^9$

(c) $y = (ax + b)^5$

4. Given $y = (16x + 3)^{-2}$, use the chain rule to find dy/dx . Then rewrite the function as $y = 1/(16x + 3)^2$ and find dy/dx by the quotient rule. Are the answers identical?

5. Given $y = 7x + 21$, find its inverse function. Then find dy/dx and dx/dy , and verify the inverse-function rule. Also verify that the graphs of the two functions bear a mirror-image relationship to each other.

6. Are the following functions strictly monotonic? For each strictly monotonic function, find dx/dy by the inverse-function rule.

(a) $y = -x^6 + 5 \quad (x > 0)$

(b) $y = 4x^5 + x^3 + 3x$

Exercise 10.5

1. Find the derivatives of:

(a) $y = e^{2t+4}$

(d) $y = 5e^{2-t^2}$

(g) $y = x^2e^{2x}$

(b) $y = e^{1-9t}$

(e) $y = e^{ax^2+bx+c}$

(h) $y = axe^{bx+c}$

(c) $y = e^{t^2+1}$

(f) $y = xe^x$

3. Find the derivatives of:

(a) $y = \ln(7t^5)$

(d) $y = 5\ln(t+1)^2$

(g) $y = \ln\left(\frac{2x}{1+x}\right)$

(b) $y = \ln(at^{10})$

(e) $y = \ln x - \ln(1+x)$

(h) $y = 5x^4 \ln x^2$

(c) $y = \ln(t+19)$

(f) $y = \ln[x(1-x)^8]$

7. Find the derivatives of the following by first taking the natural log of both sides:

$$(a) y = \frac{3x}{(x+2)(x+4)}$$

$$(b) y = (x^2 + 3) e^{x^2+1}$$

Exercise 7.4

1. Find $\partial y / \partial x_1$ and $\partial y / \partial x_2$ for each of the following functions:

$$(a) y = 2x_1^3 - 11x_1^2x_2 + 3x_2^2$$

$$(d) y = \frac{5x_1 + 3}{x_2 - 2}$$

2. Find f_x and f_y from the following:

$$(a) f(x, y) = x^2 + 5xy - y^3$$

$$(b) f(x, y) = (x^2 - 3y)(x - 2)$$

3. From the answers to Prob. 2, find $f_x(1, 2)$, the value of the partial derivative f_x when $x = 1$ and $y = 2$, for each function.

5. If the utility function of an individual takes the form

$$U = U(x_1, x_2) = (x_1 + 2)^2 (x_2 + 3)^3$$

where U is total utility, and x_1 and x_2 are the quantities of two commodities consumed:

- (a) Find the marginal-utility function of each of the two commodities.
- (b) Find the value of the marginal utility of the first commodity when 3 units of each commodity are consumed.

7. Write the gradients of the following functions:

$$(a) f(x, y, z) = x^2 + y^3 + z^4$$

$$(b) f(x, y, z) = xyz$$

Exercise 8.1

1. Find the differential dy , given:

$$(a) y = -x(x^2 + 3)$$

4. Find the point elasticity of demand, given $Q = k/P^n$, where k and n are positive constants.
 - (a) Does the elasticity depend on the price in this case?
 - (b) In the special case where $n = 1$, what is the shape of the demand curve? What is the point elasticity of demand?
6. Given $Q = 100 - 2P + 0.02Y$, where Q is quantity demanded, P is price, and Y is income, and given $P = 20$ and $Y = 5,000$, find the
 - (a) Price elasticity of demand.
 - (b) Income elasticity of demand.

Exercise 8.2

3. Find the total differential, given
 - (a) $y = \frac{x_1}{x_1 + x_2}$
4. The supply function of a certain commodity is

$$Q = a + bP^2 + R^{1/2} \quad (a < 0, b > 0) \quad [R: \text{rainfall}]$$

Find the price elasticity of supply ε_{QP} , and the rainfall elasticity of supply ε_{QR} .

5. How do the two partial elasticities in Prob. 4 vary with P and R ? In a strictly monotonic fashion (assuming positive P and R)?
6. The foreign demand for our exports X depends on the foreign income Y_f and our price level P : $X = Y_f^{1/2} + P^{-2}$. Find the partial elasticity of foreign demand for our exports with respect to our price level.
7. Find the total differential for each of the following functions:
 - (b) $U = 7x^2y^3$
 - (f) $U = (x - 3y)^3$

Exercise 8.4

2. Find the total derivative dz/dt , given

- (a) $z = x^2 - 8xy - y^3$, where $x = 3t$ and $y = 1 - t$
 - (b) $z = 7u + vt$, where $u = 2t^2$ and $v = t + 1$
 - (c) $z = f(x, y, t)$, where $x = a + bt$ and $y = c + kt$
4. Find the partial total derivatives $\partial W / \partial u$ and $\partial W / \partial v$ if
- (a) $W = ax^2 + bxy + cu$, where $x = \alpha u + \beta v$ and $y = \gamma u$
 - (b) $W = f(x_1, x_2)$, where $x_1 = 5u^2 + 3v$ and $x_2 = u - 4v^3$

Exercise 8.5

1. For each $F(x, y) = 0$, find dy/dx :
- (a) $y - 6x + 7 = 0$
 - (b) $3y + 12x + 17 = 0$
 - (c) $x^2 + 6x - 13 - y = 0$
2. For each $F(x, y) = 0$ use the implicit-function rule to find dy/dx :
- (d) $F(x, y) = 6x^3 - 3y = 0$
3. For each $F(x, y, z) = 0$ use the implicit-function rule to find $\partial y / \partial x$ and $\partial y / \partial z$:
- (a) $F(x, y, z) = x^2y^3 + z^2 + xyz = 0$

Exercise 14.2

1. Find the following:
- (a) $\int 16x^{-3} dx \quad (x \neq 0)$
 - (c) $\int (x^5 - 3x) dx$
 - (d) $\int 2e^{-2x} dx$

Exercise 14.3

1. Evaluate the following:

$$(a) \int_1^3 \frac{1}{2}x^2 dx$$

$$(e) \int_{-1}^1 (ax^2 + bx + c) dx$$

2. Evaluate the following:

$$(a) \int_1^2 e^{-2x} dx$$

$$(d) \int_e^6 \left(\frac{1}{x} + \frac{1}{1+x} \right) dx$$

5. Verify that a constant c can be equivalently expressed as a definite integral:

$$(a) c = \int_0^b \frac{c}{b} dx$$

$$(b) c = \int_0^c 1 dt$$