

Midterm 2 Sample

ECON 441: Introduction to Mathematical Economics

Instructor: Div Bhagia

Print Name: _____

This is a closed-book test. You may not use a phone or a computer.

Time allotted: 60 minutes

Total points: 20

Please show sufficient work so that the instructor can follow your work.

I understand and will uphold the ideals of academic honesty as stated in the honor code.

Signature: _____

1. (5 pts) Answer the following questions.

(a) (2 pts) Plot the following function. Is it continuous? Is it differentiable?
Justify your answer.

$$f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$$

(b) (1 pt) Find the derivative of $f(x) = \ln(x^2 + 1)$.

(c) (1 pt) Find the derivative of $f(x) = \frac{5e^x}{5x + e^x}$.

(d) (1 pt) Find the following integral: $\int_0^3 x^2 dx$

2. (5 pts) Consider a utility function for two goods x and y given by:

$$U(x, y) = x^{\frac{1}{3}}y^{\frac{2}{3}}$$

The marginal utility of each good is given by the partial derivative of the utility function with respect to that good.

- (a) (3 pts) Compute the marginal utility of x (denoted by U_x) and the marginal utility of y (denoted by U_y) for this utility function.

- (b) (1 pt) Is the marginal utility of x increasing with respect to x ? What about with respect to y ?

- (c) (1 pt) Find the total differential of U . You can denote it by dU .

3. (5 pts) Denote the demand for a good as a function of its price by $Q(p)$, and suppose that

$$\ln Q(p) = f(p)$$

- (a) (2 pts) Express the price elasticity of demand ε in terms of $f'(p)$ and p .

- (b) (2 pts) Now suppose $f(p) = 5 - 2 \ln p$. Find ε . Is demand elastic ($|\varepsilon| > 1$) or inelastic ($|\varepsilon| < 1$)?

- (c) (1 pt) In words, what does your answer in (b) say happens to the quantity demanded when the price rises by 1%?

4. (5 pts) Answer the following questions.

(a) (2 pts) Let aggregate wealth W , technology A , and population N be defined as:

$$W(t) = A(t) + N(t), \quad N(t) = ke^{st}, \quad A(t) = at$$

where k , s , and a are constants, and t represents time. Determine the total rate of change of aggregate wealth with respect to time. (You need to find the total derivative of W with respect to time.)

(b) (3 pts) A firm's marginal cost of producing output Q is

$$MC(Q) = 2Q + 4$$

and its fixed cost is 10, so that total cost at zero output is $C(0) = 10$. Find the firm's total cost function $C(Q)$. (Remember that marginal cost is the derivative of total cost.) Then, using a definite integral, find the increase in total cost when output rises from $Q = 1$ to $Q = 3$.