

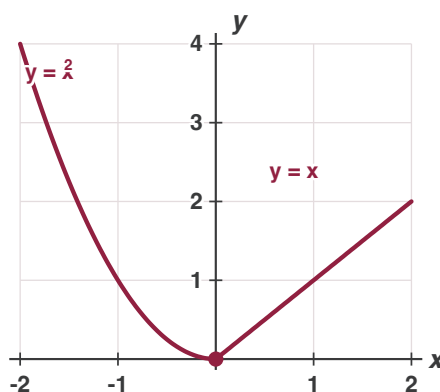
Midterm 2 Sample: Solutions

ECON 441: Introduction to Mathematical Economics

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1. (5 pts)

- (a) (2 pts) The graph is the left half of the parabola $y = x^2$ joined at the origin to the ray $y = x$:



Continuous: at $x = 0$ the left-side limit is $\lim_{x \rightarrow 0^-} x^2 = 0$, the right-side limit is $\lim_{x \rightarrow 0^+} x = 0$, and $f(0) = 0$, so the limit exists and equals $f(0)$. Away from 0 each piece is a polynomial, so f is continuous everywhere.

Not differentiable at $x = 0$: the slope from the left is $\lim_{x \rightarrow 0^-} 2x = 0$ while the slope from the right is 1. The difference quotient has different left- and right-side limits, so $f'(0)$ does not exist: the graph has a corner at the origin.

- (b) (1 pt) By the chain rule (outer function $\ln u$, inner function $u = x^2 + 1$):

$$f'(x) = \frac{1}{x^2 + 1} \cdot 2x = \frac{2x}{x^2 + 1}$$

- (c) (1 pt) By the quotient rule,

$$f'(x) = \frac{5e^x(5x + e^x) - 5e^x(5 + e^x)}{(5x + e^x)^2} = \frac{5e^x(5x - 5)}{(5x + e^x)^2} = \frac{25e^x(x - 1)}{(5x + e^x)^2}$$

(d) (1 pt)

$$\int_0^3 x^2 dx = \left[\frac{x^3}{3} \right]_0^3 = \frac{27}{3} - 0 = 9$$

2. (5 pts) $U(x, y) = x^{1/3}y^{2/3}$

(a) (3 pts)

$$U_x = \frac{\partial U}{\partial x} = \frac{1}{3}x^{-2/3}y^{2/3} \quad U_y = \frac{\partial U}{\partial y} = \frac{2}{3}x^{1/3}y^{-1/3}$$

(b) (1 pt) Differentiate U_x again:

$$\frac{\partial U_x}{\partial x} = -\frac{2}{9}x^{-5/3}y^{2/3} < 0 \quad \frac{\partial U_x}{\partial y} = \frac{2}{9}x^{-2/3}y^{-1/3} > 0$$

So the marginal utility of x is **decreasing** in x (diminishing marginal utility) and **increasing** in y .

(c) (1 pt)

$$dU = U_x dx + U_y dy = \frac{1}{3}x^{-2/3}y^{2/3} dx + \frac{2}{3}x^{1/3}y^{-1/3} dy$$

3. (5 pts) $\ln Q(p) = f(p)$ (a) (2 pts) Differentiate both sides with respect to p :

$$\frac{1}{Q(p)} \cdot \frac{dQ}{dp} = f'(p)$$

Multiply both sides by p :

$$\varepsilon = \frac{dQ}{dp} \cdot \frac{p}{Q(p)} = pf'(p)$$

(b) (2 pts) With $f(p) = 5 - 2 \ln p$, $f'(p) = -\frac{2}{p}$, so

$$\varepsilon = p \cdot \left(-\frac{2}{p} \right) = -2$$

Since $|\varepsilon| = 2 > 1$, demand is elastic, and at every price. (Equivalently, $Q = e^5 p^{-2}$: a constant-elasticity demand curve.)

(c) (1 pt) A 1% increase in the price lowers the quantity demanded by about 2%.

4. (5 pts)

(a) (2 pts) W depends on t through both A and N :

$$\frac{dW}{dt} = \frac{\partial W}{\partial A} \frac{dA}{dt} + \frac{\partial W}{\partial N} \frac{dN}{dt} = 1 \cdot a + 1 \cdot kse^{st} = a + kse^{st}$$

(b) (3 pts) Total cost is the integral of marginal cost:

$$C(Q) = \int (2Q + 4) dQ = Q^2 + 4Q + c$$

The fixed cost pins down the constant: $C(0) = c = 10$, so

$$C(Q) = Q^2 + 4Q + 10$$

The increase in cost from $Q = 1$ to $Q = 3$ is

$$\int_1^3 (2Q + 4) dQ = [Q^2 + 4Q]_1^3 = (9 + 12) - (1 + 4) = 16$$

Equivalently, $C(3) - C(1) = 31 - 15 = 16$.