

Midterm Exam 2: Help Sheet

ECON 441: Introduction to Mathematical Economics

Instructor: Div Bhagia

Limit Definition of the Derivative

$$\frac{dy}{dx} = f'(x_0) = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

Limit of a Function

The limit of a function $f(x)$ exists at $x = a$ if both the right-side and left-side limits at a are equal.

Continuity of a Function

A function $y = f(x)$ is continuous at a if the limit of $f(x)$ at a exists and is equal to the value of the function at a , i.e. $\lim_{x \rightarrow a} f(x) = f(a)$.

Product Rule

$$\frac{d}{dx} [f(x)g(x)] = f(x)g'(x) + f'(x)g(x)$$

Quotient Rule

$$\frac{d}{dx} \frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

Chain Rule

For $z = f(y)$ and $y = g(x)$:

$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx} = f'(y) g'(x)$$

Partial Derivative

If x_1 changes by Δx_1 but all other variables remain constant:

$$\frac{\Delta y}{\Delta x_1} = \frac{f(x_1 + \Delta x_1, x_2, \dots, x_n) - f(x_1, \dots, x_n)}{\Delta x_1}$$

The partial derivative of y with respect to x_i :

$$\frac{\partial y}{\partial x_i} = f_i = \lim_{\Delta x_i \rightarrow 0} \frac{\Delta y}{\Delta x_i}$$

Gradient Vector

$$\nabla f(x_1, x_2, \dots, x_n) = [f_1, f_2, \dots, f_n]^T$$

Elasticity

Percentage change in y per percentage change in x :

$$\varepsilon = \frac{dy/y}{dx/x} = \frac{dy}{dx} \cdot \frac{x}{y} = \frac{d \ln y}{d \ln x}$$

Total Differential

$$df = f_1 dx_1 + f_2 dx_2 + \dots + f_n dx_n = \sum_{i=1}^n f_i dx_i$$

Total Derivative

Total derivative with respect to x_1 :

$$\frac{df}{dx_1} = f_1 + f_2 \frac{dx_2}{dx_1} + \dots + f_n \frac{dx_n}{dx_1}$$

If x_i does not depend on x_1 then $dx_i/dx_1 = 0$.

If f does not directly depend on x_1 then $f_1 = 0$.