

Midterm Sample

ECON 441: Introduction to Mathematical Economics

Instructor: Div Bhagia

Print Name: _____

This is a closed-book test. You may not use a phone or a computer.

Time allotted: 70 minutes

Total points: 20

Please show sufficient work so that the instructor can follow your work.

I understand and will uphold the ideals of academic honesty as stated in the honor code.

Signature: _____

1. (5 pts) Answer the following questions.

(a) (1 pt) Consider a mapping $f(x)$. For two distinct values of x , x_0 and x_1 , $f(x_0) = f(x_1)$. Is f a valid function? Answer yes or no.

(b) (1 pt) Consider two sets A and B , where A is the set of all odd real numbers and B is the set of all even real numbers. What is the union of A and B ?

(c) (1 pt) Expand the following summation expression:

$$\sum_{j=1}^2 \sum_{i=1}^2 X_i Y_j =$$

(d) (1 pt) Does the inverse of the function $f : \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = x^2$ exist? If yes, find the inverse.

(e) (1 pt) Consider the following two-variable function:

$$f(x, y) = x + y$$

where $x \in (0, 1)$ and $y \in (0, 1)$. What is the range of f ?

2. (5 pts) Consider the following matrix

$$A = I - X(X^T X)^{-1} X^T$$

(a) (3 pts) Is A a square matrix? Show your work or reasoning that led you to this conclusion.

(b) (2 pts) Prove that A is idempotent i.e. $AA = A$.

3. (10 pts) Three modes of transport are available between two cities: bus, train, and plane. Denote their ticket prices by p_b , p_t , and p_a , and the corresponding quantities of tickets by q_b , q_t , and q_a . Bus and train are substitutes, as are train and plane. The demand and supply equations are as follows:

Demand Equations:

$$q_b^d = 20 - p_b + p_t$$

$$q_t^d = 36 + p_b - p_t + p_a$$

$$q_a^d = 20 + p_t - p_a$$

Supply Equations:

$$q_b^s = 12 + p_b$$

$$q_t^s = 30 + p_t$$

$$q_a^s = 16 + p_a$$

Note: At equilibrium, supply is equal to demand in each market.

- (a) (2 pts) Using the equilibrium conditions, derive three equations in the three unknown prices p_b , p_t , and p_a . Rearrange each equation so that all terms with variables are on one side and all constants are on the other.

(b) (1 pt) Express this system of equations in matrix format as

$$Av = b$$

What is A , v , and b equal to?

(c) (2 pts) Calculate the adjoint of A .

- (d) (1 pt) Calculate the determinant of A . Is A nonsingular?
- (e) (1 pt) If you premultiply A^{-1} on both sides of the equation $Av = b$, you should be able to derive an expression to solve for v . Write down this expression.
- (f) (3 pts) Using the expression in (e), solve for the equilibrium prices.