Single Variable Unconstrained Optimization

ECON 441: Introduction to Mathematical Economics — Instructor: Div Bhagia

A point x_0 is a critical/stationary point if $f'(x_0) = 0$.

Find all the critical points for the following function:

$$f(x) = x^3 - 12x^2 + 36x + 8$$

Denote the two critical points by x_1^* and x_2^*.

- What is the sign of $f'(x)$ for $x < x_1^*$ in the immediate neighborhood of x_1^*?
- What is the sign of $f'(x)$ for $x > x_1^*$ in the immediate neighborhood of x_1^*?
- What is the sign of $f'(x)$ for $x < x_2^*$ in the immediate neighborhood of x_2^*?
- What is the sign of $f'(x)$ for $x < x_2^*$ in the immediate neighborhood of x_2^*?

What are the maximum and minimum points for this function? Draw a graph for this function.

The second derivative of a function $f(x)$ is the derivative of $f'(x)$.

Find the second derivative of the above function. What is the value of $f''(x)$ at x_1^* and x_2^*?

Given the quadratic profit function

$$\pi(Q) = hQ^2 + jQ + k$$

What parameter restrictions are called for to reflect the following assumptions:

(a). If nothing is produced, the profit will be negative (because of fixed costs).

(b). The profit function is strictly concave ($\pi''(Q) < 0$ for all Q).

(c). The maximum profit occurs at a positive output level Q^*.