

Homework 6 Problems

ECON 441: Introduction to Mathematical Economics

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Exercise 10.5

1. Find the derivatives of:

(a) $y = e^{2t+4}$

(b) $y = e^{1-9t}$

(c) $y = e^{t^2+1}$

(d) $y = 5e^{2-t^2}$

(e) $y = e^{ax^2+bx+c}$

(f) $y = xe^x$

(g) $y = x^2e^{2x}$

(h) $y = axe^{bx+c}$

3. Find the derivatives of:

(a) $y = \ln(7t^5)$

(b) $y = \ln(at^\circ)$

(c) $y = \ln(t + 19)$

(d) $y = 5 \ln(t + 1)^2$

(e) $y = \ln x - \ln(1 + x)$

(f) $y = \ln [x(1 - x)^8]$

(g) $y = \ln\left(\frac{2x}{1+x}\right)$

(h) $y = 5x^4 \ln x^2$

7. Find the derivatives of the following by first taking the natural log of both sides:

(a) $y = \frac{3x}{(x+2)(x+4)}$

(b) $y = (x^2 + 3)e^{x^2+1}$

Exercise 7.4

1. Find $\partial y / \partial x_1$ and $\partial y / \partial x_2$ for each of the following functions:

(a) $y = 2x_1^3 - 11x_1^2x_2 + 3x_2^2$

(d) $y = \frac{5x_1 + 3}{x_2 - 2}$

2. Find f_x and f_y from the following:

(a) $f(x, y) = x^2 + 5xy - y^3$

(b) $f(x, y) = (x^2 - 3y)(x - 2)$

3. From the answers to Prob. 2, find $f_x(1, 2)$, the value of the partial derivative f_x when $x = 1$ and $y = 2$, for each function.

5. If the utility function of an individual takes the form

$$U = U(x_1, x_2) = (x_1 + 2)^2 (x_2 + 3)^3$$

where U is total utility, and x_1 and x_2 are the quantities of two commodities consumed:

- (a) Find the marginal-utility function of each of the two commodities.
- (b) Find the value of the marginal utility of the first commodity when 3 units of each commodity are consumed.

7. Write the gradients of the following functions:

(a) $f(x, y, z) = x^2 + y^3 + z^4$

(b) $f(x, y, z) = xyz$

Exercise 8.1

1. Find the differential dy , given:

(a) $y = -x(x^2 + 3)$

4. Find the point elasticity of demand, given $Q = k/P^n$, where k and n are positive constants.

- (a) Does the elasticity depend on the price in this case?
- (b) In the special case where $n = 1$, what is the shape of the demand curve? What is the point elasticity of demand?

6. Given $Q = 100 - 2P + 0.02Y$, where Q is quantity demanded, P is price, and Y is income, and given $P = 20$ and $Y = 5,000$, find the

- (a) Price elasticity of demand.
- (b) Income elasticity of demand.

Exercise 8.2

3. Find the total differential, given

$$(a) y = \frac{x_1}{x_1 + x_2}$$

4. The supply function of a certain commodity is

$$Q = a + bP^2 + R^{1/2} \quad (a < 0, b > 0) \quad [R: \text{rainfall}]$$

Find the price elasticity of supply ε_{QP} , and the rainfall elasticity of supply ε_{QR} .

5. How do the two partial elasticities in Prob. 4 vary with P and R ? In a strictly monotonic fashion (assuming positive P and R)?
6. The foreign demand for our exports X depends on the foreign income Y_f and our price level P : $X = Y_f^{1/2} + P^{-2}$. Find the partial elasticity of foreign demand for our exports with respect to our price level.
7. Find the total differential for each of the following functions:

$$(b) U = 7x^2y^3$$

$$(f) U = (x - 3y)^3$$

Exercise 8.4

2. Find the total derivative dz/dt , given

$$(a) z = x^2 - 8xy - y^3, \text{ where } x = 3t \text{ and } y = 1 - t$$

$$(b) z = 7u + vt, \text{ where } u = 2t^2 \text{ and } v = t + 1$$

$$(c) z = f(x, y, t), \text{ where } x = a + bt \text{ and } y = c + kt$$

4. Find the partial total derivatives $\partial W/\partial u$ and $\partial W/\partial v$ if

$$(a) W = ax^2 + bxy + cu, \text{ where } x = \alpha u + \beta v \text{ and } y = \gamma u$$

$$(b) W = f(x_1, x_2), \text{ where } x_1 = 5u^2 + 3v \text{ and } x_2 = u - 4v^3$$