Homework 4 Problems

ECON 441: Introduction to Mathematical Economics — Instructor: Div Bhagia

Exercise 5.3

1. Use the determinant $\begin{vmatrix} 4 & 0 & -1 \\ 2 & 1 & -7 \\ 3 & 3 & 9 \end{vmatrix}$ to verify the first four properties of determinants.

4. Show that when all the elements of an nth-order determinant |A| are multiplied by a number k, the result will be $k^n|A|$.

5. Calculate the determinant for the following matrices. Comment on whether the matrices are nonsingular and the rank of each matrix.

   (a) $\begin{bmatrix} 4 & 0 & 1 \\ 19 & 1 & -3 \\ 7 & 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & -2 & 1 \\ -5 & 6 & 0 \\ 7 & 0 & 3 \end{bmatrix}$

   (c) $\begin{bmatrix} 7 & -1 & 0 \\ 1 & 1 & 4 \\ 13 & -3 & -4 \end{bmatrix}$ (d) $\begin{bmatrix} -4 & 9 & 5 \\ 3 & 0 & 1 \\ 10 & 8 & 6 \end{bmatrix}$

8. Comment on the validity of the following statements:

   (a) Given any matrix A, we can always derive from it a transpose and a determinant.

   (b) Multiplying each element of an $n \times n$ determinant by 2 will double the value of that determinant.

   (c) If a square matrix A vanishes, then we can be sure that the equation system $Ax = d$ is nonsingular.

Exercise 5.4

2. Find the inverse of each of the following matrices:

(a) $A = \begin{bmatrix} 5 & 2 \\ 0 & 1 \end{bmatrix}$ (b) $B = \begin{bmatrix} -1 & 0 \\ 9 & 2 \end{bmatrix}$

(c) $C = \begin{bmatrix} 3 & 7 \\ 3 & -1 \end{bmatrix}$ (d) $D = \begin{bmatrix} 7 & 6 \\ 0 & 3 \end{bmatrix}$

3. (a) Drawing on your answers to Prob. 2, formulate a two-step rule for finding the adjoint of a given 2×2 matrix A: In the first step, indicate what should be done to the two diagonal elements of A in order to get the diagonal elements of $adj A$; in the second step, indicate what should be done to the two off-diagonal elements of A. (Warning: This rule applies only to 2×2 matrices.)

   (b) Add a third step which, in conjunction with the previous two steps, yields the 2×2 inverse matrix A^{-1}.

4. Find the inverse of each of the following matrices:

(a) $E = \begin{bmatrix} 4 & -2 & 1 \\ 7 & 3 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ (b) $F = \begin{bmatrix} 1 & -1 & 2 \\ 1 & 0 & 3 \\ 4 & 0 & 2 \end{bmatrix}$

(c) $G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ (d) $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

6. Solve the system $Ax = d$ by matrix inversion, where

(a) $$\begin{aligned} 4x + 3y &= 28 \\ 2x + 5y &= 42 \end{aligned}$$

(b) $$\begin{aligned} 4x_1 + x_2 - 5x_3 &= 8 \\ -2x_1 + 3x_2 + x_3 &= 12 \\ 3x_1 - x_2 + 4x_3 &= 5 \end{aligned}$$

7. Is it possible for a matrix to be its own inverse?

Exercise 5.5

1. Use Cramer's rule to solve the following equation systems:

(a) $3x_1 - 2x_2 = 6$
$2x_1 + x_2 = 11$

(b) $-x_1 + 3x_2 = -3$
$4x_1 - x_2 = 12$

(c) $8x_1 - 7x_2 = 9$
$x_1 + x_2 = 3$

(d) $5x_1 + 9x_2 = 14$
$7x_1 - 3x_2 = 4$

2. For each of the equation systems in Prob. 1, find the inverse of the coefficient matrix, and get the solution by the formula $x^* = A^{-1}d$.

3. Use Cramer's rule to solve the following equation systems:

(a) $8x_1 - x_2 = 16$
$2x_2 + 5x_3 = 5$
$2x_1 + 3x_3 = 7$

(d) $-x + y + z = a$
$x - y + z = b$
$x + y - z = c$