Homework 1 Solutions

ECON 441: Introduction to Mathematical Economics — Instructor: Div Bhagia

Exercise 2.3

1. (a) $\{x \in \mathbb{R} \mid x > 34\}$ or $\{x \mid x > 34\}$

   (b) $\{x \mid 8 < x < 65\}$

2. (a), (d), (f), (g), and (h) are true.

Exercise 2.4

5. We are given the function $y = 5 + 3x$ with domain $X = \{x | 1 \leq x \leq 9\}$. Note that for this function, when $x = 1$, $y = 8$ and when $x = 9$, $y = 32$. So the range for this function is:

   $$f(X) = \{y \mid 8 \leq y \leq 32\}$$

   Note: It is not always the case that extreme values of the domain correspond to extreme values of the range. For example, consider $y = x^2$ with domain $\{x \mid -2 \leq x \leq 2\}$, the range here is $\{y \mid 0 \leq y \leq 4\}$.

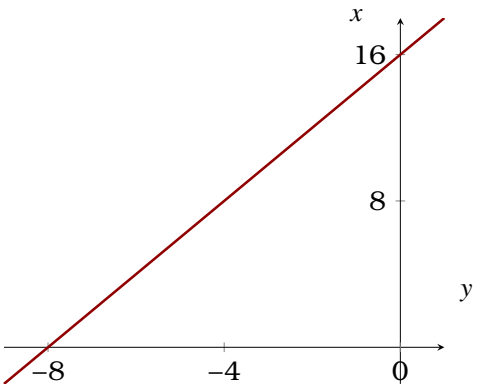
7. (a) No, (b) Yes

8. For each output, we would want to produce at the lowest cost.

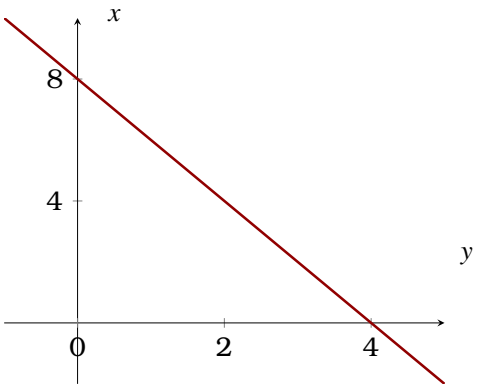
Exercise 2.5

1. Graph the following functions and find their inverse.

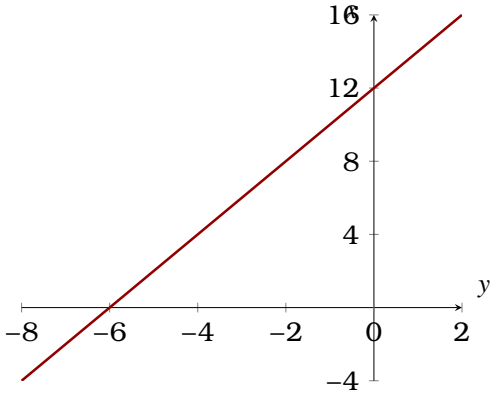
(a) $y = 16 + 2x, \quad f^{-1}(y) = \dfrac{y - 16}{2}$



(b) $y = 8 - 2x \quad f^{-1}(y) = \dfrac{8 - y}{2}$



(c) $y = 2x + 12 \quad f^{-1}(y) = \dfrac{y - 12}{2}$



Exercise 4.2

6. (a) $x_2 + x_3 + x_4 + x_5$

(b) $a_5x_5 + a_6x_6 + a_7x_7 + a_8x_8$

(c) $bx_1 + bx_2 + bx_3 + bx_4$

(d) $a_1 + a_2x + a_3x^2 + \ldots + a_nx^{n-1}$

(e) $x^2 + (x + 1)^2 + (x + 2)^2 + (x + 3)^2$

8. (a)

$$\left(\sum_{i=0}^{n} x_i\right) + x_{n+1} = x_0 + x_1 + x_2 + \ldots + x_{n+1} = \sum_{i=0}^{n+1} x_i$$

(b)

$$\begin{aligned}\sum_{j=1}^{n} ab_j y_j &= ab_1 y_1 + ab_2 y_2 + \ldots + ab_n y_n \\ &= a\left(b_1 y_1 + b_2 y_2 + \ldots + b_n y_n\right) \\ &= a \sum_{j=1}^{n} b_j y_j\end{aligned}$$

(c)

$$\begin{aligned}\sum_{j=1}^{n} \left(x_j + y_j\right) &= (x_1 + y_1) + (x_2 + y_2) + \ldots + (x_n + y_n) \\ &= x_1 + x_2 + \ldots + x_n + x_1 + y_2 + \ldots + y_n \\ &= \sum_{j=1}^{n} x_j + \sum_{j=1}^{n} y_j\end{aligned}$$

Exercise 5.1

1. (a) $q \implies p$ (b) $q \implies p$ (c) $q \iff p$

(d) $q \iff p$ (e) $q \iff p$ (f) $p \implies q$

(g) $q \implies p$