Homework 1

ECON 441: Introduction to Mathematical Economics — Instructor: Div Bhagia

Exercise 2.3

1. Write the following in set notation:
   (a) The set of all real numbers greater than 34.
   (b) The set of all real numbers greater than 8 but less than 65.

2. Given the sets $S_1 = \{2, 4, 6\}$, $S_2 = \{7, 2, 6\}$, $S_3 = \{4, 2, 6\}$, and $S_4 = \{2, 4\}$, which of the following statements are true?

   (a) $S_1 = S_3$ (b) $S_1 = \mathbb{R}$ (c) $8 \in S_2$
   (d) $3 \notin S_2$ (e) $4 \notin S_3$ (f) $S_4 \subset \mathbb{R}$
   (g) $S_1 \supset S_4$ (h) $\emptyset \subset S_2$ (i) $S_3 \supset \{1, 2\}$

   Note that $\mathbb{R}$ denotes the set of real numbers.

Exercise 2.4

5. If the domain of the function $y = 5 + 3x$ is the set $\{x | 1 \leq x \leq 9\}$, find the range of the function and express it as a set.

7. In the theory of the firm, economists consider the total cost C to be a function of the output level Q: $C = f(Q)$.
   (a) According to the definition of a function, should each cost figure be associated with a unique output level?
   (b) Should each level of output determine a unique cost figure?

8. If an output level Q_1 can be produced at a cost of C_1, then it must also be possible (by being less efficient) to produce Q_1 at a cost of $C_1 + \$1$, or $C_1 + \$2$, and so on. Thus it would seem that output Q does not uniquely determine total cost C. If so, to write $C = f(Q)$ would violate the definition of a function. How, in spite of this reasoning, would you justify the use of the function $C = f(Q)$?

Exercise 2.5

1. Graph the following functions and find their inverse functions.

   (a) $y = 16 + 2x$

   (b) $y = 8 - 2x$

   (c) $y = 2x + 12$

Exercise 4.2

6. Expand the following summation expressions:

   (a) $\sum_{i=2}^{5} x_i$ (b) $\sum_{i=5}^{8} a_i x_i$ (c) $\sum_{i=1}^{4} bx_i$

   (d) $\sum_{i=1}^{n} a_i x^{i-1}$ (e) $\sum_{i=0}^{3}(x+i)^2$

8. Show that the following are true:

   (a) $$\left(\sum_{i=0}^{n} x_i\right) + x_{n+1} = \sum_{i=0}^{n+1} x_i$$

   (b) $$\sum_{j=1}^{n} ab_j y_j = a\sum_{j=1}^{n} b_j y_j$$

   (c) $$\sum_{j=1}^{n} \left(x_j + y_j\right) = \sum_{j=1}^{n} x_j + \sum_{j=1}^{n} y_j$$

Exercise 5.1

1. In the following paired statements, let p be the first statement and q the second. Which is true for each case: $p \Rightarrow q$, $p \Leftarrow q$, or $p \Leftrightarrow q$?

   (a) It is a holiday; it is Thanksgiving Day.

   (b) A geometric figure has four sides; it is a rectangle.

   (c) Two ordered pairs (a, b) and (b, a) are equal; a is equal to b.

   (d) A number is rational; it can be expressed as a ratio of two integers.

(e) A 4×4 matrix is nonsingular; the rank of the 4×4 matrix is 4. (*skip for now*)

(f) The gasoline tank in my car is empty; I cannot start my car.

(g) The letter is returned to the sender with the marking "addressee unknown"; the sender wrote the wrong address on the envelope.