

Midterm Sample 2

ECON 441: Introduction to Mathematical Economics

Instructor: Div Bhagia

Print Name: _____

This is a closed-book test. You may not use a phone or a computer.

Time allotted: 110 minutes

Total points: 30

Please show sufficient work so that the instructor can follow your work.

I understand and will uphold the ideals of academic honesty as stated in the honor code.

Signature: _____

1. (6 pts) Answer the following questions.

(a) (1 pt) Consider a mapping $f(x)$. For two distinct values of x , x_0 and x_1 , $f(x_0) = f(x_1)$. Is f a valid function? Answer yes or no.

(b) (2 pts) Find the union and intersection for the following sets:

$$A = \{x : x \text{ is an even number}\} \quad B = \{2, 4, 8\}$$

(c) (1 pt) Consider the following two-variable function:

$$f(x, y) = x + y$$

where $x \in (0, 1)$ and $y \in (0, 1)$. What is the range of f ?

(d) (1 pt) Given a system of linear equations $Ax = b$, if $|A| = 5$, what can we say about the solution for this system of equations?

- Has no solution.
- Has a unique solution.
- Has infinitely many solutions.
- None of the above

(e) (1 pt) Is the function $y = |x|$ continuous at $x = 0$? Answer yes or no.

2. (5 pts) Consider the following matrix

$$A = I - X(X'X)^{-1}X'$$

(a) (3 pts) Is A a square matrix? Show your work or reasoning that led you to this conclusion.

(b) (2 pts) Prove that A is idempotent i.e. $AA = A$.

3. (8 pts) Consider the following system of equations:

$$x - 2z = 2$$

$$y + z = 12$$

$$x + y + z = 24$$

(a) (1 pt) Write this system of equations in matrix format i.e.,

$$Av = b$$

What is A , v , and b equal to?

(b) (2 pts) Calculate the adjoint of A .

(c) (2 pts) Calculate the determinant of A . Is A nonsingular?

(d) (1 pt) If you premultiply A^{-1} on both sides of the equation $Av = b$, you should be able to derive an expression to solve for v . Write down this expression.

(e) (2 pts) Using the expression in (d) solve for v^* .

4. (4 pts) Differentiate the following functions:

(a) $y = 3x^3 + x^2 + 4$

(b) $\frac{1}{x} + 3x^2$

(c) $\frac{x-1}{x^2+3}$

5. (5 pts) Here is a demand function:

$$Q = 100 - 0.4p$$

where $Q > 0$ is the quantity demanded and $p > 0$ is the price.

(a) Calculate the elasticity of demand ε in terms of p .

(b) What is the elasticity at $p = 50$? What about at $p = 100$? Is demand elastic ($|\varepsilon| > 1$) or inelastic ($|\varepsilon| < 1$) at these prices?

- (c) Is the elasticity monotonically decreasing or increasing with price? (Note: I suggest taking the derivative of ε with respect to p instead of guessing.)

6. (2 pts) Say we have the following relationship between income (Y), consumption (C), and saving (S).

$$Y = C + S$$

In addition, saving depends on interest rate i as follows:

$$S = g(i) + 100$$

Find the total derivative of income with respect to the interest rate.