Handout for Lecture 9

Distribution, Expectation, Variance

ECON 340: Economic Research Methods | Instructor: Div Bhagia

X is a random variable.

- Expectation of X, $\mu_X = E(X) = \sum_x x f(x)$
- Variance of X, $\sigma_X^2 = Var(X) = E[(X - \mu_X)^2] = \sum_x (x - \mu_X)^2 f(x)$
- Standard deviation of X, $\sigma_X = \sqrt{\sigma_X^2}$

If X is a random variable and $Y = a + bX$, then Y is also a random variable with

- $E(Y) = a + bE(X)$
- $Var(Y) = b^2 Var(X)$

You are at a fair and considering playing the following game — flip a coin, if you get heads, you gain \$10, else you lose \$10. Denote X as your winnings/loss from the game.

1. Find the expected value, variance, and standard deviation of X.

x	$f(x)$	$xf(x)$	$(x-\mu_X)^2$	$f(x)(x-\mu_X)^2$
10	0.5	5	10^2	50
-10	0.5	-5	$(-10)^2$	50
		0		100

Answer:

$$\mu_X = 0, \quad \sigma_X^2 = 100, \quad \sigma_X = 10$$

2. You look up and realize that you have to pay \$5 in order to play the game. So your actual winnings/loss from the game will be $Y = X - 5$. Find the expected value, variance, and standard deviation of Y.

y	$f(y)$	$yf(y)$	$(y-\mu_Y)^2$	$f(y)(y-\mu_Y)^2$
5	0.5	2.5	$(5-(-5))^2$	50
-15	0.5	-7.5	$(-15-(-5))^2$	50
		-5		100

Answer:

$$\mu_Y = -5, \quad \sigma_Y^2 = 100, \quad \sigma_Y = 10$$

Note that, $E(Y) = E(X) - 5$ and $Var(Y) = Var(X)$.

3. You see another stall offering a lower stakes game — flip a coin, if you get heads, you gain \$5, else you lose \$5. Your winnings/loss from this game will be $Z = 0.5X$. Find the expected value, variance, and standard deviation of Z.

z	$f(z)$	$zf(z)$	$(z-\mu_Z)^2$	$f(z)(z-\mu_Z)^2$
5	0.5	2.5	5^2	12.5
-5	0.5	-2.5	$(-5)^2$	12.5
		0		25

Answer:

$$\mu_Z = 0, \quad \sigma_Z^2 = 25, \quad \sigma_Z = 5$$

Note that, $E(Z) = E(X)$, $Var(Z) = (0.5)^2 Var(X)$, and $\sigma_Z = 0.5\sigma_X$.