ECON 340
Economic Research Methods

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Lecture 20
Quadratic and Log functional forms

Remember Calculus?

For a function

$$y = f(x)$$

The derivative denoted by:

$$\frac{dy}{dx} \quad or \quad f'(x)$$

captures how the value of the function changes due to a small change in x.

Rules of Differentiation

- $y = a \rightarrow \dfrac{dy}{dx} = 0$
- $y = ax \rightarrow \dfrac{dy}{dx} = a$
- $y = ax^b \rightarrow \dfrac{dy}{dx} = abx^{b-1}$
- $y = f(x) + g(x) \rightarrow \dfrac{dy}{dx} = f'(x) + g'(x)$

Examples: $y = 10, \quad y = 5x, \quad y = 8x^3, \quad y = 3x^2 + 4$

Rules of Differentiation

- Derivative of a log function:

$$y = log(x) \quad \rightarrow \quad \frac{dy}{dx} = \frac{1}{x}$$

- Chain rule:

$$z = f(y),\ y = g(x) \quad \rightarrow \quad \frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$$

Examples:

$y = 2 + 3 \cdot log(x),\ y = log(z)\ \&\ z = x^2,\ y = log(x^2),\ y = log(f(x))$

Fitting a Line

Linear relationship (with some error):

$$Y = \beta_0 + \beta_1 X + u$$

Taking the conditional expectation:

$$E(Y|X) = \beta_0 + \beta_1 X + E(u|X)$$

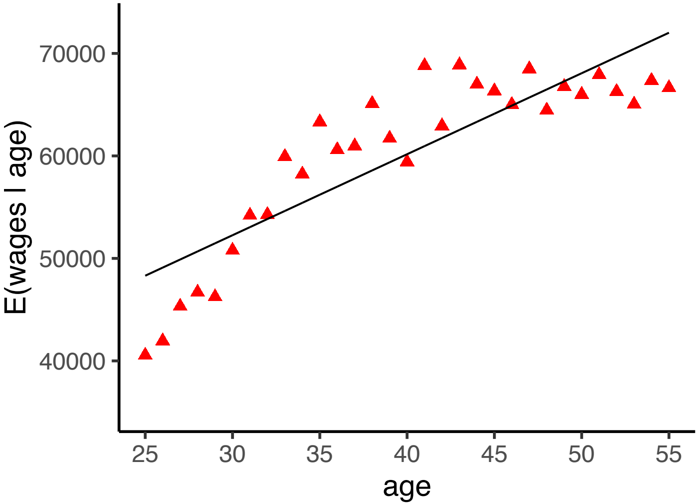
With $E(u|X) = 0$,

$$E(Y|X) = \beta_0 + \beta_1 X$$

OLS fits a linear line between average Y at each X and X.

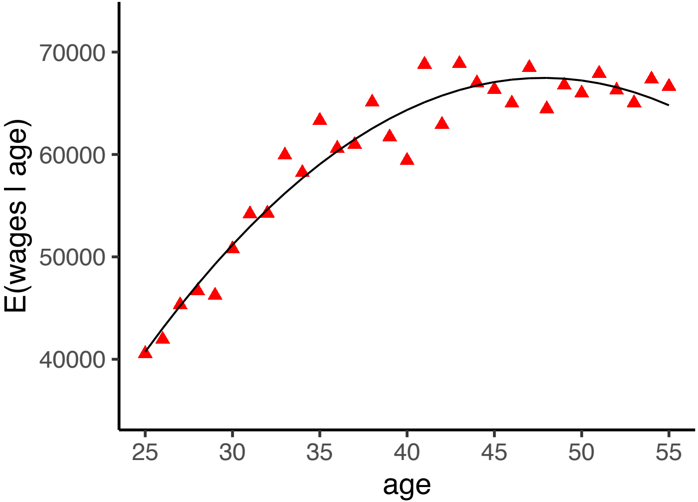
ACS Data: Wages and Age

Is the relationship really linear?



ACS Data: Wages and Age

Does this model have a better R^2?



Fitting a Line

Linear relationship:

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X$$

Take the derivative:

$$\frac{d\hat{Y}}{dX} = \hat{\beta}_1 \rightarrow d\hat{Y} = \hat{\beta}_1 dX$$

Can think of d as 'change in': One unit change in X, associated with β_1 units change in Y.

Impact of X on Y constant with X.

Fitting a Curve

Quadratic relationship:

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X + \hat{\beta}_2 X^2$$

Take the derivative:

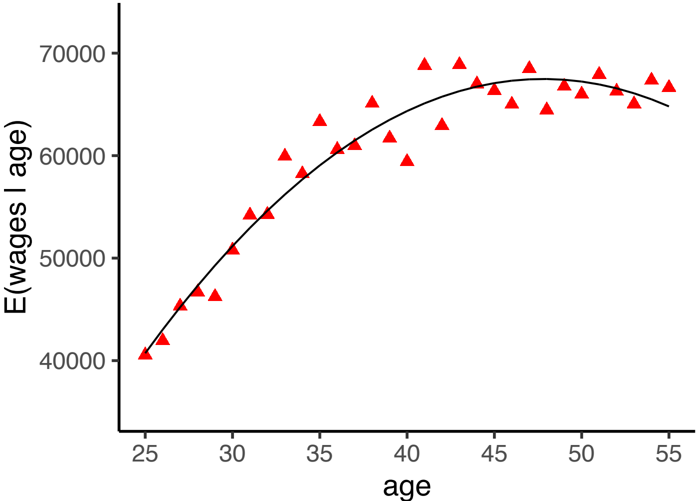
$$\frac{d\hat{Y}}{dX} = \hat{\beta}_1 + 2\hat{\beta}_2 X$$

Now the impact of X on Y changes with X.

Remember: Derivative captures the slope of the tangent line.

ACS Data: Wages and Age

$$\hat{wage} = -52207 + 4775.64 \cdot age - 49.493 \cdot age^2$$



ACS Data: Wages and Age

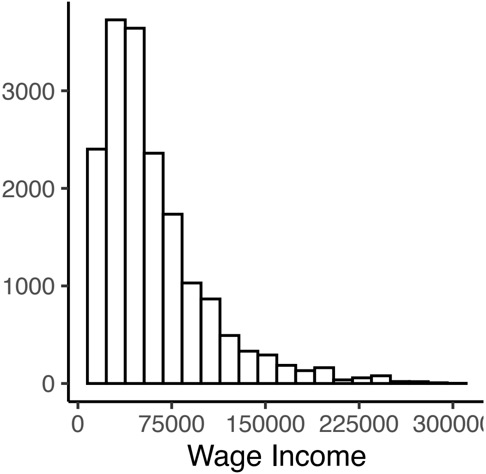
$$\hat{wage} = -52207 + 4775.64 \cdot age - 49.493 \cdot age^2$$

How does the predicted wage change going from 30 to 31?

What about going from 50 to 51?

Log Functional Forms

Sometimes we log transform a variable before fitting a model.
Useful if the data is skewed or has outliers.



Log Functional Forms

- Log-transformation leads to interpretation of regression coefficients in % changes than unit changes which can sometimes be more informative

- Can think of change in log of X as the relative change in X with respect to its original value

$$\frac{d}{dX}\log(X) = \frac{1}{X} \rightarrow d\log(X) = \frac{dX}{X}$$

In which case $100 \times d\log(X)$ represents % change in X

Log Functional Forms: Interpretation

Three possible models:

1. Level-Log: $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 log(X)$

Differentiating both left and right hand side with respect to X:

$$\frac{d\hat{Y}}{dX} = \hat{\beta}_1 \cdot \frac{1}{X} \quad \rightarrow \quad \hat{\beta}_1 = \frac{d\hat{Y}}{dX/X}$$

In which case,

$$\frac{\hat{\beta}_1}{100} = \frac{\text{unit change in } Y}{\text{\% change in } X}$$

Level-Log Model

	Wages
Intercept	$-57{,}224.83^{***}$
	(5,008.20)
Log Age	$32{,}052.27^{***}$
	(1,363.87)
Observations	17,578
R^2	0.03
Note:	*p$<$0.1; **p$<$0.05; ***p$<$0.01

1% increase in age leads to $320 increase in predicted wages.

Log Functional Forms: Interpretation

Three possible models:

2. Log-Level: $\hat{log}(Y) = \hat{\beta}_0 + \hat{\beta}_1 X$

$$\hat{\beta}_1 = \frac{1}{Y} \cdot \frac{dY}{dX} \rightarrow 100\hat{\beta}_1 = \frac{\text{\% change in } Y}{\text{unit change in } X}$$

Log-Level Model

	Log Wages
Intercept	10.31***
	(0.02)
Age	0.01***
	(0.001)
Observations	17,578
R^2	0.03
Note:	*p<0.1; **p<0.05; ***p<0.01

1 year increase in age leads to 1% increase in predicted wages.

Log Functional Forms: Interpretation

Three possible models:

3. Log-Log: $\log(\hat{Y}) = \hat{\beta}_0 + \hat{\beta}_1 \log(X)$

$$\hat{\beta}_1 = \frac{dY/Y}{dX/X} \rightarrow \hat{\beta}_1 = \frac{\%\text{ change in } Y}{\%\text{ change in } X}$$

Log-Log Model

	Log Wages
Intercept	8.99***
	(0.08)
Log Age	0.49***
	(0.02)
Observations	17,578
R^2	0.03
Note:	*p$<$0.1; **p$<$0.05; ***p$<$0.01

1% increase in age leads to 0.49% increase in predicted wages.