Handout for Lecture 13

Confidence Intervals

ECON 340: Economic Research Methods — Instructor: Div Bhagia

How to construct a confidence interval?

Known population variance: $1 - \alpha$ confidence interval for the population mean μ:

$$\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

where $z_{\alpha/2}$ is the z-value that leaves area $\alpha/2$ in the upper tail of the standard normal distribution.

Unknown population variance: $1 - \alpha$ confidence interval for the population mean μ:

$$\bar{x} \pm t_{n-1,\alpha/2} \frac{S}{\sqrt{n}}$$

where $t_{n-1,\alpha/2}$ is the t-value that leaves area $\alpha/2$ in the upper tail of the t-distribution. $n - 1$ is the degrees of freedom.

Note: Since the t distribution looks just like the standard normal for large n, for $n \geq 100$ you can continue using the standard normal table.

Question: A car manufacturer wants to estimate the mean CO_2 emissions of a new model of car. A sample of 196 cars is randomly selected and their CO_2 emissions are measured. The sample mean and standard deviation are 120 g/km and 20 g/km, respectively. Construct a 95% confidence interval for the true mean CO_2 emissions of this car model. (Note: $Pr(Z > 1.96) = 0.025$.)

Answer: We are given:

$$\begin{aligned} \bar{x} &= 120\,\text{g/km} && \text{(sample mean)} \\ S &= 20\,\text{g/km} && \text{(sample standard deviation)} \\ n &= 196 && \text{(sample size)} \end{aligned}$$

We can use the following formula to create a confidence interval:

$$\bar{x} \pm t_{n-1,\alpha/2} \frac{S}{\sqrt{n}}$$

Since we want to create a 95% confidence interval, here $1 - \alpha = 0.95$. In which case, the T statistic we want is $t_{195,0.025}$. However, since the degrees of freedom are large enough, $t_{195,0.025} \approx z_{0.025} = 1.96$. So the 95% confidence interval is given by:

$$120 \pm 1.96 \left(\frac{20}{\sqrt{196}} \right) = [117.2, 122.8]$$

Therefore, we are 95% confident that the true mean CO_2 emission of this car model is between 117.2 g/km and 122.8 g/km.